LOGICAL CONSEQUENCE: A DEFENSE OF TARKSI*

ABSTRACT. In his classic 1936 essay "On the Concept of Logical Consequence", Alfred Tarski used the notion of satisfaction to give a semantic characterization of the logical properties. Tarski is generally credited with introducing the model-theoretic characterization of the logical properties familiar to us today. However, in his book, The Concept of Logical Consequence, Etchemendy argues that Tarski's account is inadequate for quite a number of reasons, and is actually incompatible with the standard model-theoretic account. Many of his criticisms are meant to apply to the model-theoretic account as well.

In this paper, I discuss the following four critical charges that Etchemendy makes against Tarski and his account of the logical properties:

(1) (a) Tarski's account of logical consequence diverges from the standard model-theoretic account at points where the latter account gets it right.
(b) Tarski's account cannot be brought into line with the model-theoretic account, because the two are fundamentally incompatible.
(2) There are simple counterexamples (enumerated by Etchemendy) which show that Tarski's account is wrong.
(3) Tarski committed a modal fallacy when arguing that his account captures our pre-theoretical concept of logical consequence, and so obscured an essential weakness of the account.
(4) Tarski's account depends on there being a distinction between the "logical terms" and the "non-logical terms" of a language, but (according to Etchemendy) there are very simple (even first-order) languages for which no such distinction can be made.

Etchemendy's critique raises historical and philosophical questions about important foundational work. However, Etchemendy is mistaken about each of these central criticisms. In the course of justifying that claim, I give a sustained explication and defense of Tarski's account. Moreover, since I will argue that Tarski's account and the model-theoretic account really do come to the same thing, my subsequent defense of Tarski's account against Etchemendy's other attacks doubles as a defense against criticisms that would apply equally to the familiar model-theoretic account of the logical properties.

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1. INTRODUCTION

In his classic 1936 essay "On the Concept of Logical Consequence", Alfred Tarski used the notion of satisfaction – which he had earlier employed to give a precise definition of truth – to give a semantic characterization of the logical properties: logical consequence, logical truth and logical consistency. Tarski is generally credited with introducing the model-theoretic characterization of the logical properties familiar to us today,\(^1\) and so playing a key role in the early development of an incredibly fruitful area of logical research – model theory. But the status of Tarski's early work in this area has recently come under attack by John Etchemendy. In his book, *The Concept of Logical Consequence*, Etchemendy argues that Tarski's satisfactional account is inadequate for quite a number of different reasons, and far from forming the basis of the standard model-theoretic account of the logical properties, is actually incompatible with it. But Etchemendy's critique is not just aimed at an historical view held by Tarski. Many of his criticisms are meant to apply to the model-theoretic account as well.

There is a great deal of material in Etchemendy's book that wants careful examination, but in this paper, I will discuss only the following four critical charges that Etchemendy makes against Tarski and his account of the logical properties:

1. (a) Tarski's account of logical consequence diverges from the standard model-theoretic account at points where the latter account gets it right. (b) Tarski's account cannot be brought into line with the model-theoretic account, because the two are fundamentally incompatible.

2. There are simple counterexamples [enumerated by Etchemendy] which show that Tarski's account is wrong. (In fact, it is not just wrong, but wrong-headed.)

3. Tarski committed a modal fallacy when arguing that his account captures our pre-theoretic concept of logical consequence, and so obscured an essential weakness of the account.

4. Tarski's account depends on there being a distinction between the "logical terms" and the "non-logical terms" of a language, but (according to Etchemendy) there are very simple (even first-order) languages for which no such distinction can be made.

Etchemendy's critique raises historical and philosophical questions about important foundational work. However, Etchemendy is mistaken about
each of the central criticisms I mentioned. It is the purpose of this paper
to justify that claim, and to shed some needed light on Tarski’s account.
Though I will be addressing four quite different criticisms of Tarski’s
theory, my answers to these charges form a single whole—a sustained
explication and defense of Tarski’s account. Since our subject matter is
of keen historical as well as philosophical interest, I will offer an histori-
cal as well as philosophical defense. The two are not easily separated
anyway. Often enough, getting clear on what Tarski said or proposed illu-
minates just what we need to answer our philosophical concerns. More-
over, since I will argue, contra Etchemendy, that Tarski’s account and
the model-theoretic account really do come to the same thing, my sub-
sequent defense of Tarski’s account against Etchemendy’s other attacks
doubles as a defense against criticisms that would apply equally to the
familiar model-theoretic account of the logical properties.

In the next section, I give an overview and careful formulation of
Tarski’s 1936 account of logical consequence. In the ensuing four sections
of the paper, I discuss each of the four Etchemendy criticisms listed
above, and show that each is mistaken.²

2. TARSKI’S ACCOUNT

To unravel what has gone wrong in Etchemendy’s arguments, I will first
present Tarski’s 1936 account of logical consequence. The aim here is
to give a precise formulation of the theory which is faithful to Tarski’s
original and allows us to introduce subsidiary concepts which will ease
further exposition. But I also wish to make certain aspects, as well as the
overall structure, of the account more perspicuous. A certain amount of
technical detail is unavoidable in carrying out these tasks, but our efforts
will be considerably rewarded by ease of subsequent exposition.

In Tarski’s conception, the logical properties are properties of senten-
ces and sets of sentences. They are also formal properties, that is,
sentences of a given language have or lack these properties in virtue
of the forms of those sentences. Let L be a meaningful language freely
generated by a set of syntactic, expression-forming operations from a
base set of symbols, S, which set is further distinguished into a set of
variables, VarL, and the remainder, SymL = S − VarL.³ Let τ be a
function which partitions S into semantic categories.⁴ It is important to
note that L is to be a meaningful language. The issues that concern us
do not even arise with respect to uninterpreted or semi-interpreted lan-
guages. For example, since the sentences of a semi-interpreted language
are, strictly speaking, neither true nor false, it would make no sense to
talk of the logical consequence relation for that language being truth preserving.

Tarski's idea is to extend the application of his notion of satisfaction by a sequence from truth to the logical properties. To evaluate the logical properties of a sentence, he will replace each of the 'non-logical terms' in the sentence with an extra-linguistic variable of appropriate type (in a sense to be specified), and then consider the "satisfaction behavior" of the resulting formula. It will be useful for our technical and philosophical purpose to articulate this Tarskian strategy by first introducing a notion - that of term function - in terms of which we can give the Tarskian definitions of the logical properties, and which, in itself, subsumes most of the technical business in Tarski's account.

Let $Z$ be a set of symbols - our extra-linguistic variables - which is disjoint from $S$.

DEFINITION. $F$ is a term function for $L$ if $F$ is a total 1-1 function from $S$ to $S \cup Z$ such that

(a) for any $\nu \in \text{Var} L$, $F(\nu) = \nu$, and

(b) for any $\zeta \in \text{Sym} L$, either $F(\zeta) = \zeta$ or $F(\zeta) \in Z$, and

(c) for all $z \in Z$, $\exists \zeta \in S$ such that $F(\zeta) = z$.

Call the terms of $L$ on which a term function $F$ is the identify function, the $F$ terms of $L$, and all other terms, non-$F$ terms of $L$. When the relativization to $F$ is understood, these classes are also known as the fixed and non-fixed terms of $L$. To anticipate, these classes of terms may be called the logical and non-logical terms of $L$ for certain choices of $F$.

For the following, let $\varphi$ be a sentence of $L$, $\Gamma$ a set of sentences of $L$, and $F$ a term function for $L$.

DEFINITION. Let $F[\varphi]$ be the sentential function whose value for a sentence $\varphi$ is the result of replacing symbol $\zeta$ in $\varphi$ with the symbol $F(\zeta)$, for every symbol $\zeta$ in $\varphi$. Also let $F[\Gamma] = \{F[\psi] \mid \psi \in \Gamma\}$.

So, $F[\varphi]$ is a formula (in a hybrid language) in which all the non-fixed terms have been replaced by extra-linguistic variables. Now, why should the satisfaction behavior of such formulas tell us anything about, say, the logical truth of the original sentences? Here is one way to motivate the idea. Suppose that a formula $F[\varphi]$ is satisfied by every sequence. Then it can be shown that the original sentence $\varphi$ is also satisfied by every sequence. According to Tarski's semantic definition of truth, this is just the circumstance in which $\varphi$ is true. Thus, the satisfaction behavior of this formula $F[\varphi]$ is enough to determine that $\varphi$ is true. Since $F[\varphi]$ only
retains the fixed terms and grammatical form of \( \varphi \), if the fixed terms may justly be called "logical terms", it is plausible to say that \( \varphi \) is "true in virtue of its logical form". In other words, we might take the indicated determination relation as one way of fleshing out the intuition behind the informal claim that logically true sentences are true in virtue of their logical form.

In (Tarski, 1936), Tarski did not give a careful definition of the sort of sequences his account needs, nor does he define the notion of satisfaction. Instead, he adverts to (Tarski, 1933) where both these notions were treated in detail. Following Tarski's lead, then, we may define sequences in the following way.\(^7\) We suppose that, for each semantic category determined by \( \tau \), there is a class of appropriate semantic values for terms of that category - its semantic value class. For example, for individual constants of the language, the semantic value class might be individuals. For the \( n \)-place predicates of the language, the semantic value class might be classes of \( n \)-tuples of individuals, etc. For convenience, we may write \( \tau (\zeta) \) to indicate the semantic value class of a term, \( \zeta \).

**DEFINITION***. For language \( L \), term function \( F \), and letting \( f \) be any fixed 1-1 mappings of \( Z \cup \text{Var} L \) onto the natural numbers, we shall say that \( s \) is a *sequence* (relative to \( L, F, f \)) if \( df s \) is a function with domain the natural numbers, and such that for all \( \chi \in Z \cup \text{Var} L \), \( s(f(\chi)) \) is in the semantic value class of the symbol \( F^{-1}(\chi) \).

Our choice of \( f \) is understood to remain fixed throughout the discussion, and so we will ordinarily suppress this parameter for ease of exposition. In discussing sequences, we also suppress the parameters \( L \) and \( F \) where no confusion will arise. Intuitively, using the metaphor of sequences, the value \( s(n) \) is the occupant of the \( n \)th position of the sequence \( s \).

I will not give a definition of the companion notion of *satisfaction by a sequence*. For this we would need to choose a particular meaningful language.\(^8\) In addition to the notion of a sequence \( s \) satisfying a formula \( F[\varphi] \), it is convenient to speak of a sequence \( s \) satisfying a set of formulas \( F[\Gamma] \) where this just means that \( s \) satisfies \( F[\psi] \) for each \( \psi \in \Gamma \).

As a final precursor, we introduce the following technical notions.

**DEFINITION.** \( \varphi \) is an *\( F \)-consequence* (in \( L \)) of \( \Gamma \) if \( df \) every sequence, \( s \), that satisfies \( F[\Gamma] \) also satisfies \( F[\varphi] \).

**DEFINITION.** \( \varphi \) is *\( F \)-true* (in \( L \)) if \( df \) every sequence \( s \) satisfies \( F[\varphi] \).

**DEFINITION.** \( \Gamma \) is *\( F \)-consistent* (in \( L \)) if \( df \) some sequence \( s \) satisfies \( F[\Gamma] \).
These are not characterizations of our target logical properties. The notions defined so far are relative to a choice of term function (in addition to being language relative). To get a characterization of the logical properties for a language, it is clear that we must choose an appropriate term function. Tarski expected that the appropriate term function would be such that the fixed terms and non-fixed terms would correspond to the then extant, intuitive distinction between the logical and non-logical terms of familiar logical languages. But he does not in his essay give any general criterion for what counts as a logical term, though he acknowledges the need for such. At any rate, it is evident that Tarski’s account presupposes the existence of an appropriate term function for each language in the broad class of languages $\mathcal{K}$ that Tarski’s account is to cover, say, those treated in (Tarski, 1933). Call these desired functions, *logical term functions*. So, Tarski’s theory presupposes the existence of logical term functions, but doesn’t offer any kind of account of logical terms. This makes the theory incomplete, as Tarski (1936, p. 418) acknowledges. To complete the theory would require filling this gap with a *principled* treatment of the notion of a logical term, something Tarski did not come to until 1966, when he proposed a model-theoretic invariance criterion for logical termhood.⁹ We can now represent Tarski’s account (for a given language, $L$, in class $\mathcal{K}$) as an existence claim, three definitions and a claim of adequacy.

**CLAIM [Existence].** There exists a logical term function (for $L$).

**DEFINITION.** $\varphi$ is a *Tarskian consequence* (in $L$) of $\Gamma$ (i.e. $\Gamma \models_{T} \varphi$) if$\text{df}$ there is a logical term function $F$ for $L$ such that $\varphi$ is an $F$-consequence of $\Gamma$.

**DEFINITION.** $\varphi$ is a *Tarskian logical truth* (in $L$) if$\text{df}$ there is a logical term function $F$ for $L$ such that $\varphi$ is $F$-true.

**DEFINITION.** $\Gamma$ is *Tarskian logically consistent* (in $L$) if$\text{df}$ there is a logical term function $F$ for $L$ such that $\Gamma$ is $F$-consistent.

Note, I am assuming here that the (missing) definition of logical term function will yield the result that any two logical term functions for a language will always yield the same $F$-consequences (etc.). We need this in order to ensure that the three terms just introduced are well-defined (though their extensions might be empty, if the existence claim was false).

Having thus introduced special terms for Tarski’s defined notions, we must enter into the account a claim to the effect that these defined terms
bear an appropriate relationship to the pre-theoretic notions which are the target of the account. However, stating a precise adequacy condition on Tarski's behalf is not without its problems. At first blush, the following condition seems to fit the bill.

CLAIM [Co-extensiveness]. For all sentences \( \varphi \) of \( L \), and all sets of sentences \( \Gamma \) of \( L \),

(i) \( \varphi \) is a Tarskian consequence of \( \Gamma \) iff \( \varphi \) is a logical consequence \( \Gamma \),
(ii) \( \varphi \) is a Tarskian logical truth iff \( \varphi \) is logically true, and
(iii) \( \Gamma \) is Tarskian logically consistent iff \( \Gamma \) is logically consistent.

Where the italicized terms represent the target pre-theoretic notions.

However, just like in the case of truth, Tarski did not think of himself as straight-forwardly defining a notion in its pre-theoretic usage.

[Attempts to make the pre-theoretic notion precise] have been confronted with the difficulties that usually present themselves in such cases. With respect to the clarity of its content the common concept of consequence is in no way superior to other concepts of everyday language. Its extension is not sharply bounded and its usage fluctuates. Any attempt to bring into harmony all possible vague, sometimes contradictory, tendencies which are connected with the use of this concept, is certainly doomed to failure. (Tarski, 1936, p. 409.)

Then, the adequacy claim formulated above cannot, strictly speaking, be what Tarski had in mind. In fact, while the possibility of inconsistency receives only passing mention in this passage, it is likely that Tarski is committed, by extension from his view of closely-related semantic notions such as truth and satisfaction, to the view that the notion of logical consequence as applied to natural languages is incoherent. And, indeed, the class of languages \( K \) to which Tarski’s account is directed is a class of formalized and semantically open, interpreted languages.\(^{10}\)

So, what claim did Tarski wish to make for his definition? Tarski’s approach to this sort of project could be characterized this way. He would first identify some central characteristics of his target notion, and then would formulate a precise adequacy condition which could intuitively be seen to capture the informally identified characteristics. Once he had a precise condition, the object for him was to define a term which satisfied the adequacy condition. There was then no more question for him of how the defined notion related back to the original characterizations, insofar as he had already distilled what he thought could be made clear when he formulated his adequacy condition.

Tarski clearly followed this procedure with regard to truth. The notion he defines is beholden to two adequacy conditions, one formal (having
to do with avoidance of the semantic paradoxes), and the other the well-known Convention T (that the definition provably entails all T-sentences for the language). Tarski did something similar for logical consequence, though there is nothing available in the case of consequence that is quite so compelling as Convention T, and the formulation of a precise adequacy condition is more problematic. He identified two central characteristics of logical consequence. Roughly, these were that logical consequence is (a) truth preserving, and (b) formal-logical, i.e. that sentences stood in this relation due to their logical form, and more particularly, that standing in this relation was independent of the sense of the non-logical terms of the sentences.\(^{11}\) Tarski then formulated a precise condition incorporating these two characteristics. This condition, [Substitution Condition], is set out carefully below in Section 3.2. Tarski's definition provably satisfies this condition. However, the condition is only a necessary one (\textit{vis-à-vis} the two characteristics he identified), as Tarski himself points out. So, Tarski does not tell us that a definition of logical consequence is to be considered \textit{adequate} just in case it provably entails [Substitution Condition]. Instead, the adequacy condition that seems to be operative in (Tarski, 1936) is much more directly tied to the two characteristics of logical consequence that Tarski identified.

CLAIM [Adequacy]. A definition of a predicate, \(\zeta\), for "logical consequence in L" is \textit{adequate} just in case

(i) \(\zeta\) is provably truth preserving (cf. Appendix B, Corollary C), and  
(ii) \(\zeta\) is (provably) formal-logical (cf. Appendix C, Theorem F).

Though Tarski does not present this as a precise adequacy condition \textit{as such}, he does lay claim to both these results when offering support for his account. A careful discussion of Tarski's claim to these results will be reserved until Section 5.3 and Section 5.5. Why did Tarski not put forward this adequacy condition \textit{as such}? I suspect that Tarski did not have a precise way of formulating the condition which met with his scruples. A full spelling out of clause (ii) seems to require quantification over all extensions of the given language. For Tarski, the trouble with this as a formal condition is that, in order for it to say what we intend, the metalanguage in which it is expressed must stand outside the Tarskian hierarchy. This would have been unacceptable to Tarski (and should be worrisome for us), because, even though his discursive remarks throughout the 1933 and 1936 papers are in just such a metalanguage, Tarski went to great pains to ensure that the language employed in all his definitions and proofs could be construed as expressed in some language
within the hierarchy. This is crucial to Tarski’s strategy for avoiding the paradoxes.

The end result is that, so far as formal conditions go, we do not find anything more than a partial adequacy condition in the 1936 paper. This is not, in itself, a difficulty for the account, it simply means that Tarski did not have everything he would have needed to make the adequacy of the account admit of definitive proof. For Tarski, it would not have admitted of proof for other reasons, anyway. Since Tarski had no account of logical terms, he was not in a position to offer any kind of proof of the [Existence] claim. For these reasons, there could be no question for him of definitively establishing the correctness of his account.12

Tarski spoke of the material adequacy of his definition. We know that this is not, for Tarski, the claim [Co-extensiveness]. Rather, we may think of material adequacy, in this context, as co-extensiveness with a carefully circumscribed usage of ‘logical consequence’. And we may think of [Adequacy] as articulating the boundaries of that usage. This understanding of Tarski’s use of ‘materially adequate’ is parallel to the way that term is used in (Tarski, 1933; 1944).

While the claim, [Co-extensiveness], suggested earlier, is at best a simplification of any condition that Tarski would have accepted, it will prove useful in our discussion, because Etchemendy does take it as a measure of correctness of the account whether it is co-extensive (perhaps even necessarily co-extensive) with the pre-theoretic notion (cf. Section 5). With this synoptic view of Tarski’s account of logical properties, let us turn to the first of Etchemendy’s criticisms on our list.13

3. TARSKIAND AND MODEL-THEORETIC CONSEQUENCE

It is widely thought that the account of the logical properties that Tarski introduced just is the model-theoretic account. Etchemendy makes the surprising claim that the two are in fact incompatible, and in a way detrimental to Tarski’s account. But before we turn to the subtleties of Etchemendy’s argument for this, there is a more readily evident source of discrepancy between the two accounts. It seems that Tarski wanted his account to classify as consequence-related things that we know are not so classified by the model-theoretic account. In particular, Tarski wanted his account to validate arithmetical inferences from \( \{ \varphi(1), \varphi(2), \varphi(3), \ldots \} \) to for all natural numbers \( x, \varphi(x) \). So, his account was intended to yield a consequence relation such that, for arithmetic languages in \( \mathcal{K} \), closure under this relation would always produce \( \omega \)-complete sets, defined as follows:
DEFINITION. For an arithmetic language $L$ (with a zero constant, '0', a predicate, 'N', for natural number, and 'S' for the successor relation), we say that a set of sentences, $A$, is $\omega$-complete if, for any formula $\varphi[x]$, if $\varphi[S_n0] \in A$ for every natural number $n$, then $\forall x(Nx \rightarrow \varphi x) \in A$.\textsuperscript{14}

By extension, call a consequence relation $\omega$-complete just in case the closure under that relation of any set of sentences $A$ is an $\omega$-complete set.

In this sense, Tarski thought that the relation of logical consequence was $\omega$-complete, and he sought an account that would preserve this feature. No proof-theoretic treatment of consequence has this feature, but neither does the model-theoretic account of consequence.\textsuperscript{15} Closing off under standard model-theoretic consequence does not always yield $\omega$-complete sets. That is why we have, e.g., non-standard models of first-order arithmetic.

While Tarski does not show us how his account gives us the desired $\omega$-completeness, presumably he thinks it does. Etchemendy makes the plausible suggestion that Tarski got his consequence relation to be $\omega$-complete by treating the arithmetical terms ('0', 'N', and 'S') of arithmetical languages as logical terms (Etchemendy, 1988, p. 73).\textsuperscript{16} In any event, as Tarski would have it, his account does not come to quite the same thing as the model-theoretic account, though we have not at all established that this sort of divergence would be a defect in Tarski's account.

So, by pursuing Tarski's intentions, we will get a divergence in extension between Tarskian and model-theoretic consequence. Nonetheless, this sort of discrepancy seems rather innocuous, since it wholly depends on what terms we choose to take as fixed. Thus, if it were deemed prudent, or were forced by our final account of logical terms, we could bring Tarski's account into line with the model-theoretic account simply by making an appropriate choice of term function. In fact, I take it that Tarski's 1966 account of logical notions would do just this by classifying terms like '0', 'N', and 'S' as non-logical (Tarski, 1986). Let us follow suit. In this case, we have no reason so far to suppose that Tarski's account and the model-theoretic account do not come to the same thing, and certainly have none for thinking them incompatible. In fact, as Gila Sher (1996) has argued, we need not even give up $\omega$-completeness, so long as we formulate the inductive inferences in a second-order way.

3.1. The Divergence Argument

Be that as it may, Etchemendy argues on separate grounds that, without some modification, Tarski's account and the model-theoretic account are
not equivalent for any selection of fixed terms. In fact, Etchemendy’s charge goes considerably further than this. He maintains that (a) Tarski’s account diverges from the model-theoretic account in places where the latter is getting the right answer, and that (b) Tarski’s account cannot be consistently refined to come into line with the model-theoretic account. In short, Tarski’s account is said to be incompatible with the model-theoretic account, and in a way that shows Tarski’s account to be inadequate.\footnote{To begin to see where the divergence is supposed to enter, consider the following argument. On the standard model-theoretic account sentences are, for example, logically true just in case they are true in every model, where a model is a structure that specifies a domain of discourse, as well as extensions for the constants and predicates of the language, which are built from the elements of the domain of discourse. Yet, Tarski’s sequences have no special provision for the specification of a domain of discourse, and this leads to trouble, as we can show with the following example. On the plausible assumption that ‘∃’, ‘=’, and ‘¬’ are proper logical terms, \(F[\exists x \exists y \ x \neq y]\) is identical to \(\forall x \exists y \ x \neq y\). On Tarski’s account, ‘\(\exists x \exists y \ x \neq y\)’ is a logical truth just in case \(F[\exists x \exists y \ x \neq y]\) is satisfied by every sequence, or, what comes to the same thing in this case, iff ‘\(\exists x \exists y \ x \neq y\)’ is satisfied by every sequence. But also, by Tarski’s definition of truth, a sentence is true just in case it is satisfied by every sequence. Thus, it follows that if ‘\(\exists x \exists y \ x \neq y\)’ is true (which no doubt it is), then Tarski’s account will mistakenly pronounce it a logical truth. A careful demonstration of this is given in Appendix A. Of course, ‘\(\exists x \exists y \ x \neq y\)’ fails to be a logical truth on the standard model theoretic account, because there are models with singleton domain in which the sentence is false. So, it looks as though Tarski’s account both (a) diverges from the model-theoretic account, and (b) gets the wrong answers. To get it right, evaluation of satisfaction by sequences would have to somehow take into account a varying a domain of discourse with respect to which quantifiers would be evaluated, and from which the other elements of the sequence are built. This might be done by letting the sequences themselves specify a domain of discourse. Call such sequences domain-relativized.}

Let me first address myself to the historical issue this argument raises. Etchemendy takes the lack of any discussion of relativizing at face value, and so sees a deficit in Tarski’s account. But it is puzzling to think that Tarski just missed the need for domain relativization. The 1936 paper which concerns us here is given in very general terms, and, atypically, nothing is worked out in detail. So, the historical record leaves room for — nay, invites — speculation on whether Tarski had some sort of domain
relativization in mind. Corcoran (1973, p. 70) speculates that Tarski did not, but his grounds for thinking this seem to be only the fact that Tarski was "working in a framework of an interpreted language having a fixed domain of discourse," and, as far as I can see, this fact is quite irrelevant to the issue, since it simply doesn't tend to point in either direction. Wilfrid Hodges (1986), on the other hand, speculates that there is no mention of relativizing simply because Tarski's essay was transcribed from a "talk addressed to philosophers, and [Tarski] didn't think philosophers would be interested." It is prima facie quite possible that the matter of domain relativization was suppressed as a matter only of detail.

I have identified two pieces of textual evidence that bear on this issue. First, Tarski makes a simple claim in his essay that would be false, if indeed he were making tacit use of domain-relativized sequences. Tarski claims that, if one treated all the terms of a language as logical terms (i.e. let F be the identify function on the terms of the language), "the concept of formal consequence would then coincide with that of material consequence. The sentence $\varphi$ would in this case follow [formally and materially] from the class of sentences $\Gamma$ if either $\varphi$ were true or at least one sentence of the class $\Gamma$ were false" (p. 419). If sequences somehow specify a domain of discourse with respect to which quantifiers are evaluated, i.e., if sequences act like models in the modern sense, then this claim of Tarski's is simply false. The following example, borrowed from Gila Sher, serves to illustrate the point. 'There are exactly two things' is a material consequence of 'There is exactly one thing', because the latter sentence is false. However, under current assumptions, 'There are exactly two things' is not a formal consequence of 'There is exactly one thing', because there are domain-relativized sequences with singleton domains, and each such sequence satisfies the latter sentence, but not the former. Thus, Tarski's claim is in error, if domain relativization is assumed. On the other hand, Tarski's claim comes out true, if we employ unrelativized sequences (assuming as we are here that this forces a fixed domain for quantifier evaluation).

So, Tarski either made the mistake of missing the need for domain-relativized sequences, or he made the mistake of thinking that logical and material consequence would collapse when all the terms of a language were fixed. Are there grounds for attributing one of these errors over the other? Sher says that charity dictates that we attribute the lesser error, pending further evidence to the contrary. And it is clear that the latter is the lesser error, because making the first mistake would vitiate Tarski's entire account, while the second would not. I think this is probably the
right attitude to take, but if the scales need further tipping, we can actually
do a bit more.

Everything about the way Tarski proceeds in his 1936 paper suggests
that he is setting things up so that he may simply make use of his (1933)
work on truth, and satisfaction by a sequence. And it is to that work that
Tarski adverts in lieu of any detailed explanation of the notion of satis-
faction by a sequence in the 1936 paper. This might, at first, appear more
damning than not, because Tarski is working with unrelativized sequences
in the 1933 paper on truth. However, though it is not often remarked,
there is a section of that classic essay in which Tarski introduces some-
thing which does all the work of domain-relativized sequences. Tarski’s
purpose in doing so is to define the notion of “truth with respect to an
individual domain”, from which he shows he can once again define truth
\textit{simpliciter}. Rather than change the notion of sequence, however, Tarski
chooses to introduce a ternary satisfaction relation, which, for Tarski’s
language of the calculus of classes, is as follows.

\textbf{DEFINITION.} A sequence, \( s \), \textit{satisfies} formula \( \varphi \) \textit{in the individual do-
main}, \( D \), if \( \text{df} \) \( D \) is a class of individuals, \( s \) is a sequence of subclasses
of \( D \), and

\begin{enumerate}
\item[(i)] if \( \varphi \) is atomic, then \( s \) satisfies \( \varphi \) \textit{simpliciter},
\item[(ii)] if \( \varphi \) is of the form \( \forall \psi \neg \theta \) for some \( \psi \), then \( s \) does not satisfy \( \psi \) in
the individual domain \( D \),
\item[(iii)] if \( \varphi \) is of the form \( \forall \psi \lor \theta \) for some \( \psi \) and \( \theta \), then \( s \) satisfies either
\( \psi \) or \( \theta \) in the individual domain \( D \), and
\item[(iv)] if \( \varphi \) is of the form \( \exists \chi \psi \) for some formula \( \psi \) and variable \( \chi \), then
there is a \( \chi \)-variant, \( s_\chi \), of \( s \) such that \( s_\chi \) satisfies \( \psi \) in the individual
domain \( D \).
\end{enumerate}

The “domain” introduced in this definition differs in only one way from
a modern model-theoretic domain of discourse. The quantifiers do not
range over the elements of \( D \) itself, but range instead over the elements
of the power set of \( D \). But this is merely a language-specific convenience
that Tarski avails himself of. There is absolutely no \textit{functional} difference
between domains of discourse in the usual sense, and Tarski’s indi-
vidual domains. Nor is there any functional difference between these and
domains of discourse which have been incorporated right into sequences
themselves. Thus, Tarski had already seen how to relativize to domains
of discourse in the 1933 paper. The domain-relativized satisfaction rela-
tion does not take center stage in that essay, because the relativization
to domains does not play any appreciable role in the case of truth \textit{sim-
pllciter}. Tarski shows how to define truth \textit{simpliciter} on the basis of this
3.2. The Incompatibility Argument

Etchemendy has an argument to the effect that domain-relativization is incompatible with Tarski's account, regardless of whether Tarski tacitly employed domain relativized sequences. This is of more than historical interest, since we have not yet seen any necessary divergence between Tarski's account and the model-theoretic account. In order to set the stage for Etchemendy's incompatibility argument, it will be helpful to say a bit more about how Tarski makes the case for his account.

Tarski begins by articulating two intuitive characteristics of logical consequence. First, if a sentence \( \varphi \) is a consequence of a set of sentences \( \Gamma' \), then "it can never happen that both the class \( \Gamma' \) consists only of true sentences and the sentence \( \varphi \) is false," i.e., the consequence relation is truth preserving. Second, a logical consequence relation must be "uniquely determined by the form of the sentences between which it holds" [pp. 414–415], i.e., logical consequence is a formal relation. Accordingly, Tarski formulates a precise condition, Substitution Condition1, which has both a truth-preservational and a formal component which is said to be a necessary condition on logical consequence.28 Two preliminary definitions help us formulate Tarski's condition below.

**DEFINITION.** A function, \( \rho \), is a *respectful replacement function* for \( L \) if \( \rho \) is a total function from the set of non-logical terms of \( L \) into itself such that for every non-logical symbol, \( \zeta \), of \( L \), \( \rho(\zeta) \) and \( \zeta \) are of the same semantic category. That is, \( \tau(\zeta) = \tau(\rho(\zeta)) \) for each non-logical \( \zeta \).

For sentence, \( \varphi \), we write \('\rho(\varphi)\'\) for the result of replacing symbol \( \zeta \) in \( \varphi \) with the symbol \( \rho(\zeta) \), for every symbol \( \zeta \) in \( \varphi \). For a set of sentences, \( \Gamma \), we write \('\rho(\Gamma)\'\) for \( \{\rho(\psi): \psi \in \Gamma\} \).
DEFINITION. The replacement class of sentence $\varphi$, is $\{\rho(\varphi): \rho$ is a respectful replacement function for $L\}$.

CONDITION [Substitution Condition$_1$]. For all respectful replacement functions, $\rho$, if all the sentences in $\rho(\Gamma)$ are true, then the sentence $\rho(\varphi)$ is true.

CLAIM [Substitutional Necessity$_1$]. The Substitution Condition$_1$ is a necessary condition for $\varphi$ to be a logical consequence of $\Gamma$. That is, if a sentence $\varphi$ is a logical consequence of the set of sentences $\Gamma$, then for all respectful replacement functions, $\rho$, if all the sentences in $\rho(\Gamma)$ are true, then the sentence $\rho(\varphi)$ is true.

It will be useful to note that a similar claim for logical truth follows from this.\(^{29}\)

CONDITION [Substitution Condition$_2$]. For all respectful replacement functions, $\rho$, the sentence $\rho(\varphi)$ is true.

CLAIM [Substitutional Necessity$_2$]. The Substitution Condition$_2$ is a necessary condition for the logical truth of $\varphi$. That is, if a sentence $\varphi$ is logically true, then for all respectful replacement functions, $\rho$, the sentence $\rho(\varphi)$ is true.

Tarski argues that, while the Substitution Condition$_1$ is a necessary condition for logical consequence, it is not also a sufficient condition. That is, we cannot maintain that

\[(EB_1) \text{ The Substitution Condition}_1 \text{ is a necessary and sufficient condition for } \varphi \text{ to be a logical consequence of } \Gamma; \text{ that is, a sentence } \varphi \text{ is a logical consequence of a set of sentences } \Gamma \text{ iff for all respectful replacement functions, } \rho, \text{ if all the sentences in } \rho(\Gamma) \text{ are true, then the sentence } \rho(\varphi) \text{ is true.}\]

The reason is this. First, notice that [EB$_1$] entails a parallel claim about logical truth.

\[(EB_2) \text{ The Substitution Condition}_2 \text{ is a necessary and sufficient condition for } \varphi \text{ to be a logical truth; that is, a sentence } \varphi \text{ is a logical truth iff for all respectful replacement functions, } \rho, \text{ the sentence } \rho(\varphi) \text{ is true.}\]

Now, the variety of sentences in the replacement class of a sentence $\varphi$ is dependent on what we can get by substituting terms in for the non-logical
terms of \( \varphi \), and this in turn is limited by the expressive capacity of the language we are working with. If the language is very limited, \([EB_2]\) can lead to wrong results. Here is a simple example. Suppose that our language has only two predicates, has-a-heart and has-kidneys. Now, as a matter of fact, all creatures that have a heart also have kidneys, and all creatures that have kidneys have a heart.\(^{30}\) That is, the following sentence is true:

\[
[hk] \quad \text{For all } x, \text{ } x \text{ has-a-heart iff } x \text{ has-kidneys.}
\]

Letting this be our \( \varphi \), and assuming the universal quantifier and biconditional count as logical terms, we note that a respectful replacement function, \( \rho \), could yield only the following four sentences as value for \( \rho(\varphi) \):

- For all \( x \), \( x \) has-a-heart iff \( x \) has-kidneys.
- For all \( x \), \( x \) has-a-heart iff \( x \) has-a-heart.
- For all \( x \), \( x \) has-kidneys iff \( x \) has-a-heart.
- For all \( x \), \( x \) has-kidneys iff \( x \) has-kidneys.

And since each of these sentences is true, it follows that \( \varphi \) will satisfy Substitution Condition\(_2\). Following claim \([EB_2]\), that would make \([hk]\) a logical truth. This is surely wrong. Thus, we cannot accept \([EB_2]\), and by extension, cannot accept \([EB_1]\) either.

Though it is somewhat tangential to our main line, a historical remark is in order here. Etchemendy wrongfully attributes the claim \([EB_1]\) to Bernard Bolzano (1837, Section 147), and most of Etchemendy’s claims about Bolzano are thereby vitiated. The trouble is that Bolzano’s account is about \textit{propositions}, not sentences, and the substitutions he considers are substitutions of objective \textit{ideas}, not linguistic terms. This difference turns out to be crucial, because such an account does not suffer from the same insufficiency just noted for the Substitution Condition\(_1\). This is because Bolzano’s \textit{ideas} are not limited by the expressive capacities of any language (Section 49.2). Surprisingly, Etchemendy knows he is flouting the historical record, and this is revealed in a footnote.\(^{31}\) Etchemendy tells us there that this historical indiscretion is “to facilitate comparison of Tarski and Bolzano.” How misrepresenting Bolzano helps to compare his views with Tarski’s is a mystery, especially since the historical truth seems to be that Bolzano’s account is in effect much closer to Tarski’s than is Etchemendy’s \([EB_1]\).\(^{32}\)

Let us pick up the main thread, by returning to our example of a non-logical truth, \([hk]\), that meets Substitution Condition\(_2\). We might chalk
this failure up to the fact that the language we were working with did not provide a rich enough collection of predicates for substituting. [hk] satisfied Substitution Condition$_2$ with respect to the meager language we set up, but would certainly have failed to in simple extensions of that language which included additional predicates of the right sort. However, while logical consequence and logical truth are formal, and hence, language-relative notions, they are not relative in this way to the expressive power of a language. In particular, we expect that if two meaningful languages differ only by terms which do not appear in a given sentence (argument) common to both languages, the logical status of that sentence (argument) should be the same in both languages. Etchemendy subsumes a family of intuitions of this sort under the following principle:

[Persistence Principle] The logical properties should persist through all permissible expansions (and contractions) of the language. (p. 31.)

We are now in a position to state clearly Etchemendy’s argument against Tarski. Recall that domain relativization is required in order for Tarski’s account to have any chance of getting the right answers. Etchemendy argues that this requirement cannot be met on pain of inconsistency. He considers and rejects, in turn, each of three obvious approaches to domain relativization (pp. 66–77). One way to get domain relativization is to treat quantifier words as non-logical. If quantifiers are non-logical, then there is a position in Tarski’s sequences which holds some kind of semantic value assignment for those quantifiers, and this semantic value might be made to play the role of a domain of discourse. However, Etchemendy shows that this strategy will not work, because it would certainly violate Tarski’s adequacy condition, Substitutional Necessity. To see this, consider the following case. ‘Something is busy’ is a logical consequence of ‘Bettina is busy’, so by Substitutional Necessity, we should have, for all respectful replacement functions, $\rho$, if $\rho(\text{‘Bettina is busy’})$ is true, then $\rho(\text{‘Something is busy’})$ is true. Now, since ‘everything’ and ‘something’ are of the same semantic category and are both being treated as non-logical under current assumptions, there is a respectful replacement function, $\rho^*$, which maps every non-logical term to itself, except $\rho^*(\text{‘something’})=\text{‘everything’}$. Easily, it might happen that $\rho^*(\text{‘Bettina is busy’})$, i.e. ‘Bettina is busy’, is true, yet $\rho^*(\text{‘Something is busy’})$, i.e. ‘Everything is busy’, is false. Thus, Substitutional Necessity is violated.

So, as a second strategy of domain relativization, Etchemendy proposes that ‘something’ be treated as composed of two logically separate particles, ‘some’ and ‘thing’, where the former is to be a logical term
and the latter a non-logical term.\textsuperscript{34} That way there must still be a place in every Tarski sequence which assigns a semantic value to ‘thing’, and which may be pressed into service as a domain of discourse; yet we do not get the same sort of violation of Substitutional Necessity as we saw above.\textsuperscript{35} A domain of discourse must have the function not only of fixing the range of the quantifiers, it also must put a limit on the semantic value assignments to the rest of the terms of the language. So, to introduce into Tarski’s sequences the sort of domain relativization we are looking for, we will also have to impose what Etchemendy calls \textit{cross-term restrictions}. The logical properties are then to be defined in terms of a special subclass of sequences.

**DEFINITION.** A sequence \( s \) is \textit{domain-restricted}\textsuperscript{1} if \( s \) every element in the sequence \( s \) is “built out of” the elements of (that set which is) the semantic value \( s \) assigns to ‘thing’.

So, for example, Tarskian logical truth would now be characterized via an appropriately revised definition of \( \Gamma \)-true.

**DEFINITION.** A sentence \( \varphi \) is \textit{\( F \)-true}\textsuperscript{1} if \( \varphi \) every domain-restricted\textsuperscript{1} sequence satisfies \( \Gamma[\varphi] \).

**DEFINITION.** \( \varphi \) is a \textit{Tarskian logical truth}\textsuperscript{1} if \( \varphi \) there is a logical term function \( \Gamma \) such that \( \varphi \) is \( \Gamma \)-true\textsuperscript{1}.

Now, however, Etchemendy will argue that getting domain relativization in this way is, after all, inconsistent with two fundamental principles of Tarski’s account: Substitutional Necessity, and the Persistence Principle (pp. 69–70). Instead of directly relating Etchemendy’s own argument, I am going to offer a closely-related incompatibility argument which is rather simpler than Etchemendy’s. The argument I will present has the virtue of showing that Etchemendy is wrong to think that the problem he sees essentially involves the Persistence Principle. That will be demonstrated by the fact that via my modified version of Etchemendy’s argument, we can show that the current strategy for getting domain-relativization is already inconsistent with Tarski’s adequacy condition, Substitutional Necessity.

Consider a very simple language, \( L \), which is a fragment of English including only the following terms: one name (‘Abe Lincoln’), one predicate (‘was president’), one logical connective (‘if’–‘then’), one logical quantifier particle (‘some’), two non-logical common nouns (‘thing’ and ‘dog’). The sentence

\[ [1] \quad \text{If Abe Lincoln was president, then some thing was president.} \]
is a Tarskian logical truth, on current assumptions. But Substitutional Necessity does not hold in L, because, while [1] is a true sentence, the following false sentence is in its replacement class.

[2] If Abe Lincoln was president, then some dog was president.

Thus, this second proposed strategy for getting domain-relativity into Tarski’s account is not compatible with Substitutional Necessity either. But, recall, Etchemendy wants to conclude that domain-relativization, however managed within the basic framework of the Tarskian account, will be incompatible with Substitutional Necessity and the Persistence Principle (cf. Note 17). To make an inductively strong case for that conclusion, all of the odds-on candidates for introducing domain relativity must be considered, at least. Yet, it is evident that they have not been. There are at least two obvious alternatives, one patterned after the domain-relativization Tarski introduced in (Tarski, 1933), and one patterned after modern model-theoretic structures. The former strategy is to introduce a ternary satisfaction relation (“s satisfies φ relative to domain D”). The latter strategy is to introduce a special position in sequences, say, a 0th position, the occupant of which would determine the domain for the sequence. Let us follow Etchemendy and consider only this latter approach, though I should stress that Etchemendy’s criticism of this approach and my rejoinder could be worked through for the Tarskian strategy equally well.

We introduce a new kind of sequence – a d-sequence – which has a domain-specifying 0th position. For a given d-sequence, s, we will denote by ‘Dom(s)’ the occupant of this 0th position of s. Tarskian logical truth would now be characterized by a revised definition of F-true based on this new kind of sequence.

**DEFINITION.** A d-sequence s is domain-restricted if every element in (a non-0 position) in s is “built out of” the elements of the set Dom(s).

**DEFINITION.** A sentence φ is F-true if every domain-restricted d-sequence satisfies F[φ].

**DEFINITION.** A sentence φ is a Tarskian logical truth if there is a logical term function F such that φ is F-true.

Etchemendy’s argument against this approach is straightforward enough. He believes the difference between this strategy and the quantifier-splitting one is merely cosmetic; the very same argument is said to work against this approach. Once again, the claim goes, we can produce sentences
deemed logically true by the new account, such as [1], which nonetheless have false sentences, such as [2], in their replacement class. Thus, Substitutional Necessity is violated in the very same way as before (Etchemendy, pp. 67–69).

On the contrary, I shall argue that Etchemendy has erred here, and that the current strategy for domain relativization is quite beyond the reach of Etchemendy’s argument. To see this, notice that Etchemendy’s “some dog” argument can only work when ‘something’ is treated as consisting of a logical and a non-logical particle. It is this that put [2] in the replacement class of [1], and so led to the violation of Substitutional Necessity. Yet, the quantifier terms were only split into two particles on Etchemendy’s suggestion as part of that previously considered and failed strategy to domain-relativize sequences. Having now introduced a position in sequences to house a domain of discourse, why should we treat ‘something’ as anything other than a unitary, logical term, as has been customary? Apparently, Etchemendy believes that it would be inappropriate to declare ‘something’ a logical term in any case, because its contribution to the truth values of sentences still differs as radically from \(d\)-sequence to \(d\)-sequence as that of ‘Abe Lincoln’ or ‘was president’ (p. 76).

To back this claim, Etchemendy notes that the semantic values that \(d\)-sequences can assign to these terms are all constrained in the same way by the domain of the \(d\)-sequence.

If we were to list all the various constraints imposed by our new “implicit parameter” [the domain of the \(d\)-sequence], we would have to include among them the following three: ‘Abe Lincoln’ can name any individual we please, so long as the individual is a member of the domain of discourse; ‘was president’ can have any extension we please, so long as that set is a subset of the domain of discourse; and finally ‘something’ can be restricted however we please, so long as the restriction set is identical to the domain of discourse (p. 76).

If this is meant by Etchemendy in such a way as to presuppose that \(d\)-sequences assign a “restriction set” or any kind of semantic value at all to the quantifiers, then it makes a false presupposition – one that would be appropriate in the first two proposals for obtaining domain relativization, but not this third proposal. The current proposal is to treat quantifiers as fixed terms, and so \(d\)-sequences would, by definition, not make assignments to ‘something’ any more than they do to ‘and’. Etchemendy must show what is wrong with doing this.

Perhaps, though, his point is something like this: the range with respect to which the quantifiers are in effect evaluated varies from \(d\)-sequence to \(d\)-sequence just like the extensions do with respect to which predicates are in effect evaluated. Yet, if this were grounds for
declaring quantifiers non-logical, it would likewise be grounds for declaring virtually every term non-logical. For example, the extension with respect to which 'and' is in effect evaluated also varies from $d$-sequence to $d$-sequence.\footnote{Mutatis mutandis, the same would apply to all other terms we ordinarily classify as logical. If we limited our class of logical terms accordingly, evidently little or nothing would be F-true$_2$, and nothing would be an F-consequence$_2$ of anything but itself. So, in effect, Etchemendy is imposing on the classification of logical terms (i.e. the choice of F) a constraint that is guaranteed to give us the wrong results. But surely any constraint on logical terms which eliminates all of the terms which have been pre-theoretically identified as logical is a poor constraint indeed.}

Even so, however, I think it is not hard to see a quite natural principle underlying Etchemendy's words.

A logical term must contribute uniformly to the satisfaction conditions of formulas in a way that non-logical terms do not.

No doubt there is some truth to this. One might even see in it something of Tarski's latter day identification of logical notions as those which are invariant under certain permutations (Tarski, 1986). But, true as the above principle may be when properly construed, it simply cannot be cashed out in terms of non-variation across $d$-sequences of extension or range. It is not easy to formulate a general and precise condition for a term's being logical. Tarski himself didn't know how this could be done in 1936. If the problem continued to look intractable, and all the principled conditions on offer failed as spectacularly as the constraint imposed by Etchemendy, then perhaps we could muster a healthy skepticism about the prospects for this distinction. But it is not so. There is an extant criterion for logical terms, namely the one mentioned above which Tarski offered in 1966. This criterion bids fair to succeed on both counts — classifying terms in a way that comports well with the pre-theoretic distinction, and delivering what are prima facie logical term functions in the sense needed for Tarski's theory. I conclude that Etchemendy has no good argument against treating quantifiers as logical terms, and employing domain-restricted$_2$ $d$-sequences. It should be noted that this is, in effect, the approach which is built into the model-theoretic account. Thus, if we construe Tarski's account in this way, as I will for the remainder of this essay, then we achieve a full reconciliation of the Tarskian and model-theoretic accounts.

In sum, we have found good historical reasons to reject Etchemendy's divergence argument, because it depends on the uncharitable assumption that Tarski was not even tacitly employing domain-relativized sequences.
More importantly, Etchemendy's argument to the effect that the Tarskian and model-theoretic accounts are incompatible is without foundation. The two accounts are quite readily reconciled. Our arguments thus far have shown that the only point where we may have to part with the historical Tarski is in the matter of \( \omega \)-completeness. Thus, our investigations restore the received view that in 1936 Tarski did introduce what is, in its essentials, the model-theoretic account of the logical properties with which we are familiar today. But also note that, because of this convergence, all of Etchemendy's ahistorical criticisms must now be understood to apply equally to the Tarskian and model-theoretic accounts. It will be good to bear this in mind, since more of our remaining discussion can be seen in this light to be of more than historical interest.

4. ON PUTATIVE COUNTEREXAMPLES TO TARKSI'S ACCOUNT

Etchemendy (1990, Chapter 8) gives a series of putative counterexamples to Tarski's account. I will review each of the (as I count them) five counterexamples, and reduce them to a single problem. I will then propose what I take to be the right way of handling that problem.

Etchemendy's counterexamples all take advantage of a single aspect of Tarski's account. Recall that the Substitution Condition failed to be sufficient for logical consequence because what substitutions were available was constrained by the size of the language and the variety of terms available in it. The condition for Tarskian logical consequence, namely \( F \)-consequence (for suitable \( F \)), may be vulnerable to a semantic version of this problem. What sequences are available for the satisfaction relation is constrained by the size of the universe and the variety of things available in it. As far as I know, this worry for Tarski's account was first raised by John Corcoran (1972, p. 43; 1973, p. 70). Etchemendy's counterexamples, if successful, would vindicate Corcoran's suspicions. Let us turn then to a consideration of these putative counterexamples.

Case #1. Suppose that the logical terms of the language include \( \exists \), ' = ', and the usual connectives. Some sentences of the following form are true.

\[
\exists x \exists y (x \neq y) \quad \text{i.e., there are at least two things.}
\]

\[
\exists x \exists y \exists z (x \neq y \neq z \neq x) \quad \text{i.e., there are at least three things.}
\]

etc...

Since each of these sentences, \( \psi \), is composed of nothing but logical terms, \( F[\psi] = \psi \) for any logical term function, \( F \). So, if \( \psi \) is true (i.e.
satisfied by all sequences), then $F[\psi]$ is satisfied by all sequences, and hence $\psi$ is a Tarskian logical truth. Since some of the above sentences are indeed true, some are Tarskian logical truths. But it is intuitively obvious that none of these sentences is really a logical truth. (Cf. p. 111.)

In fact, we have already done the work to undermine this case. Its success as a counterexample depends entirely on the presupposition that the existential quantifier has a fixed range of evaluation across all sequences, because it is only under these circumstances that the truth of a sentence implies that it satisfies all sequences. In short, this case presupposes that Tarski's account must be understood as lacking domain-relativization. I argued in Section 3 of this essay, that this is a mistake. If we employ $d$-sequences and take Tarskian logical truth$_2$ for logical truth, then the above argument fails. So, as it happens, the putative counterexample is actually avoided by adopting the standard model-theoretic treatment of quantification.

Case #2. Suppose that the logical terms of the language include ‘$\exists$', ‘=’, and the usual connectives. Suppose that the universe is in fact finite. Then, some sentences of the following form are true.

\[ \neg \exists x \exists y (x \neq y) \quad \text{i.e., there is at most one thing.} \]

\[ \neg \exists x \exists y \exists z (x \neq y \neq z \neq x) \quad \text{i.e., there are at most two things.} \]

etc...

To avoid the flaw in the first case, let us allow that the quantifiers are evaluated with respect to the variable domains of $d$-sequences. Nonetheless, the domain of any $d$-sequence cannot exceed the totality of things that exist. But then, since the universe is finite by hypothesis, infinitely many of these sentences will be satisfied by all $d$-sequences, and since these sentences are composed of nothing but logical terms, it follows that they are Tarskian logical truths$_2$. But again, it is intuitively obvious that none of them is really a logical truth. (Cf. p. 113.)

This case avoids the pitfall of the first case, but there is also a rather painless way of handling it. We need only deny that the universe is finite. In fact, by allowing the usual specification of a language, or the set-theoretic apparatus of sequences, we have prima facie already committed ourselves to an infinity of objects (albeit abstract ones). So, it looks like there would be some kind of pragmatic incoherence in these circumstances for one who insisted on a finite universe tout court. Unless, of course, they were in a position to offer us a finitist nominalization of
our metatheoretical talk, but in that case they would seem to have handed the Tarskian a saving alternative, namely a way to represent infinities with finitely many things. So, the Tarskian could simply adapt.

Now, Etchemendy will still object that an analysis of the logical properties should not presuppose substantive facts about the world, such as its being infinite. This objection surfaces in our very next case.

**Case #3.** As in Case #2, suppose that the logical terms of the language include ‘∃’, ‘=’, and the usual connectives. But this time allow that the universe is infinite. Then, none of the following sentences is true and none satisfied by all d-sequences, and so none is a Tarskian logical truth. So far, so good.

\[-\exists x \forall y (x \neq y)\]  
\[-\exists x \exists y \exists z (x \neq y \neq z \neq x)\]

i.e., there is at most one thing.  
i.e., there are at most two things.  
 etc...

Nonetheless, the universe could have been finite. And had the universe been finite, some of these sentences would have satisfied all the d-sequences that then existed, and thus would have met the conditions for Tarskian logical truth. But this is problematic for two reasons. (1) Logical truth should be a subspecies of necessary truth. But then, though some of the above sentences could have been Tarskian logical truths, none could have been logical truths, since all of them are false. (2) Logical truths should be true in virtue of the meanings of their logical terms, and should not depend for their truth on extra-logical facts. But the fact that some of these sentences would have met Tarski's conditions had the universe been finite shows that some sentences meet (or fail to meet) Tarski's conditions only in virtue of an extra-logical fact, namely the infinity of things in the universe. (Cf. pp. 113–114.)

Regarding the first charge associated with this case, one could respond by denying the premise that the universe could have been finite. Perhaps this could be maintained, for example, by a mathematical platonist who is already committed to the existence of infinitely many necessary existents. I will take a rather different line. Notice that the argument which is the basis of the first charge rests on the following claim.

Tarski's account commits us to the claim that had the universe been finite, certain sentences would have met the conditions for Tarskian logical truth, and thus would have been logical truths.
I maintain that this presupposition of Etchemendy’s argument is false. However, a full discussion of the matter will be postponed until after all the examples have been discussed.

Regarding the second charge associated with this case, I think Tarski would have regarded talk about what a sentence was “true in virtue of” as unclear and rightly so, but if we must speak this way, it seems to me that the charge depends on not distinguishing between what makes a sentence true, and what it takes to show that a sentence meets Tarski’s condition. There is a false presupposition lurking here which enables Etchemendy to run these two together, and which will likewise be brought to light in the next section.

Some of Etchemendy’s cases are designed to show that we cannot avoid the difficulties presented by the first three cases by treating identity as a non-logical term. These cases follow the pattern of Cases #2 and #3, but do not involve sentences with identity.

Case #2a. Suppose that the logical terms of the language include ‘∃’, and the usual connectives. Suppose that the universe is in fact finite.

\[-(\forall xyz(xRyRz \rightarrow xRz) \land \forall x \neg (xRx) \land \forall y \exists x(xRy))\]

Since the universe is finite the formula that results from applying F to this sentence will be satisfied by every d-sequence, and so the displayed sentence counts as a Tarskian logical truth, but it is intuitively clear that it is not a logical truth, whether or not the finiteness assumption is true. (Cf. pp. 117–119.)

Here again, we may deny that the universe is in fact finite, or, if need be, pirate whatever explanation the finitist offers of all our good infinitary talk. The next case is supposed to forestall this move.

Case #3a. As in Case #2a, suppose that the logical terms of the language include ‘∃’, and the usual connectives. But also allow that the universe is infinite.

\[-(\forall xyz(xRyRz \rightarrow xRz) \land \forall x \neg (xRx) \land \forall y \exists x(xRy))\]

Under current assumptions, some d-sequences do not satisfy the formula that results from applying F to this sentence. However, that formula would have been satisfied by every d-sequence had the universe been finite, and so the displayed sentence would have been a Tarskian logical truth. But it is intuitively clear, that this sentence is not, nor could it have been, a logical truth. (Cf. p. 118.)
I propose to handle this case just like Case #3, because this case also presupposes [3].

Finally, to show that the trouble for the Tarski account is widespread, Etchemendy also offers this case.

*Case #3b.* Suppose that the logical terms of the language include the usual connectives. Suppose also that the semantic value class for the category of \(n\)-ary predicates is a class of properties not extensions. (A meager conception of property is assumed, whereby not every set of tuples corresponds to a property.)

\[(Pa \rightarrow Pb)\]

If the world had been completely homogeneous, then the displayed sentence would have met the conditions for Tarskian logical truth, but it is intuitively clear that the above sentence is not a logical truth. (Cf. p. 121.)

An obvious response to this is to reject the use of properties as semantic values of predicates. But such a move is not forced on us, since the argument for this case also mistakenly presupposes the counterfactual claim

Tarski’s account commits us to the claim that had the universe been homogeneous, certain sentences would have met the conditions for Tarskian logical truth, and thus would have been logical truths

which is so closely related to [3] that this case can be handled of a piece with Cases #3 and #3a.

4.1. *The Modal Status of Tarski’s Account*

We have seen that Etchemendy’s Cases #1, #2, and #2a can be handled by (i) counting the usual connectives and identity as logical terms, (ii) counting the existential quantifier as a logical term (in the manner discussed in the previous section), and (iii) maintaining that the universe is in fact infinite (or deploying our favorite nominalization of our infinitary talk, in this case set-theoretic talk). The remaining cases, [3], [3a], and #3b, each presuppose either claim [3] or [4]. I will argue that these claims are false. The fault lies in an assumption common to both, namely that Tarski’s account commits us to the following counterfactual.

For any sentence, had that sentence met the conditions for *Tarskian logical truth*, that sentence would have been a logical truth.
Such a counterfactual would only be underwritten by Tarski's account if that account had a certain modal force which, it appears, it was not intended to have. There are good reasons to suppose that Tarski's aim was to give a materially adequate characterization of logical consequence. However, Etchemendy maintains that Tarski's goal was to produce a conceptual analysis of logical consequence. That would mean that Tarski's condition should get the right answers in any conceptually possible circumstance, and so underwrite [cf]. But that is just what Etchemendy's 3x cases seem to show Tarski's condition does not do, because its success depends on certain (putatively) contingent facts. Earlier, Corcoran (1972, p. 44) was likewise led to the conclusion that the account's dependency on contingent facts, e.g. the size and variety of available domains of quantification, undermined its status as conceptual analysis. But what leads Etchemendy to the conclusion that Tarski's goal was conceptual analysis in the first place?

Tarski emphasizes that his goal is to give a definition of consequence that captures, as far as possible, the "essentials" of our ordinary concept. His goal is, in other words, analysis. (Etchemendy, 1988, pp. 70-71.)

I gather that Etchemendy is advertiring to the following remark by Tarski. After raising problems for any syntactic treatment of consequence, Tarski says,

In order to obtain the proper concept of consequence, which is close in essentials to the common concept, we must resort to quite different methods and apply quite different conceptual apparatus in defining it. (1936, p. 413.)

Perhaps Etchemendy highlights the word 'essentials', because he supposes that to define a notion "close in essentials" to the ordinary notion of consequence would be to "capture the essence" of that concept, i.e. to give a conceptual analysis. A merely materially adequate account of consequence would not do this. However, it is instructive to take note of the response Tarski gave to philosophers who complained that his account of truth went no further than material adequacy.

I have heard it remarked that the formal definition of truth has nothing to do with "the philosophical problem of truth". However, nobody has ever pointed out to me in an intelligible way just what this problem is. I have been informed in this connection that my definition, though it states necessary and sufficient conditions for a sentence to be true, does not grasp the "essence" of this concept. Since I have never been able to understand what the "essence" of a concept is, I must be excused from discussing this point any longer. (Tarski, 1944, p. 61.)

This suggests, to me at any rate, that it would be a mistake to read too much into the occurrence of the word 'essentials' (or its Polish equivalent) in the passage to which Etchemendy alludes. Certainly, if the cor-
rectness of Tarski’s account hinges on it, Etchemendy’s reading is at best uncharitable.

There are other reasons to think that Etchemendy has misread Tarski. The fact is that in his 1936 essay Tarski tells us that what he is pursuing is a materially adequate definition of logical consequence. Toward the end of the essay, having given his definition, and prior to discussing some open questions about the account, he says

I am not at all of the opinion that in the result of the above discussion the problem of a materially adequate definition of the concept of consequence has been completely solved. On the contrary, I still see several open questions... (Tarski, 1936, p. 418, italics mine.)

This certainly sounds like Tarski is telling us what he takes the project to be – it is the project of giving a materially adequate definition of logical consequence.

I would like to bring one further consideration forward. It is clear that what Tarski is doing in the 1936 essay is applying to the case of the logical properties the satisfictional apparatus he developed in his work on truth. One of the key features of logical consequence that Tarski wishes to capture is truth preservation. Since he has already given a precise, materially adequate definition of truth, it is quite natural to think that one could use the same conceptual apparatus to formulate conditions for truth preservation. However, it is quite clear that Tarski’s account of truth does not have the status of conceptual analysis. The conventional wisdom, to which Etchemendy subscribes, is that analysis was not Tarski’s aim there, rather he proposed only a materially adequate and formally correct definition of truth. But then, it seems to me that it would be quite surprising if, when Tarski turned to consequence, he was looking for an analysis. If he had only material adequacy with his satisfictional technique in the truth case, how startling to think he could be getting a conceptual analysis with those same techniques in the case of logical consequence. To bring the point home, consider the case of logical truth. It makes sense to employ the same conceptual apparatus used in one’s account of truth to characterize logical truth, since logical truth is some species of truth, presumably. But if your appeal to that conceptual apparatus (in this case the notion of satisfaction by a sequence) yielded an account of truth that is only materially adequate, surely you could not expect anything stronger of an account of logical truth likewise based on the same apparatus!

For all these reasons, it seems reasonable to suppose that the Tarskian account of logical consequence is only intended to be a materially adequate account. Let us return, then, to Etchemendy’s 3x cases to see how they are effected by this judgment. These remaining unresolved cases
for any sentence, had that sentence met the conditions for

Tarski's logical truth; that sentence would have been a logical truth.

It should now be clear that the Tarskian account does not commit us to this at all. A materially adequate account need not support such counterfactuals. Etchemendy’s belief that Tarski’s account does is based on the mistaken belief that the account is supposed to be a conceptual analysis. I conclude that none of Etchemendy’s cases proves to be a genuine counterexample to Tarski’s account. The same conclusion may be drawn *mutatis mutandis* for the standard model-theoretic account of the logical properties. Note also that we can readily answer a complaint which I tabled earlier. To handle some of Etchemendy’s examples, we assumed that there are infinitely many things, and Etchemendy objects to this assumption on the grounds that it is illegitimate in giving an analysis to rely on such an empirical fact (if it is an empirical fact). Evidently, this complaint, too, relies on treating Tarski’s account as a conceptual analysis. Stewart Shapiro sounds the rejoinder this way:

We don’t need to claim that the axiom of infinity is true on logical grounds, since we are out to *model* the intuitive notion of consequence, not to give some sort of philosophical analysis of it. We don’t have to prove on *logical grounds alone* that our model is a good one. In defending our account, we use whatever resources are available to us. (Shapiro, forthcoming.)

In fact, Charles Chihara thinks that, even if Tarski’s account was to be put forward as a philosophical analysis, it is not at all clear what the basis for Etchemendy’s complaint is supposed to be.

No reason has been given to think that [an account of logical consequence that is “dependent on completely non-logical facts”] must be defective. Is an analysis of some geometrical phenomenon defective just because it relies upon some non-geometrical facts, say facts of arithmetic?... Will it be objected that the output of [such an] account remains dependent on completely non-geometrical truths? If so, why is that an objection that should cause any concern? (Chihara, forthcoming.)

I close this section with one further argument that Etchemendy gives, which is closely related to the foregoing. In the course of giving his counterexamples, Etchemendy suggests an interesting argument (p. 116), involving the doctrine of finitism, which would seem to show that Tarski’s account must be mistaken even if [cf] is rejected, and the universe is infinite. Etchemendy claims that, even if we grant that there are infinitely
many objects, Tarski’s account must be wrong because it would render finitism, i.e., the claim that

\[ [5] \quad \text{there are only finitely many things.} \]

\textit{inconsistent} with the claim that

\[ [6] \quad \text{no sentence of the form ‘there are less than } n \text{ things’ is logically true.} \]

I suppose that Etchemendy’s reasoning must go something like this. Assume

\[ [7] \quad \text{Tarski’s account is correct, and hence logical truth and Tarskian logical truth are at least extensionally equivalent.} \]

Now suppose that [5] holds. Then there is some finite \( \mathbf{n} \) such that there are less than \( \mathbf{n} \) things. That would mean that every \( d \)-sequence has a domain of less than \( \mathbf{n} \) elements. Since the sentence \( ‘\text{there are less than } \mathbf{n} \text{ things’} \) contains only logical terms,\(^{42}\) it follows that every \( d \)-sequence will satisfy it, and, hence, that sentence will be a Tarskian logical truth. Applying [7] and generalizing, we conclude that there is an \( \mathbf{n} \) such that \( ‘\text{there are less than } \mathbf{n} \text{ things’} \) is a logical truth, and this contradicts [6]. Note, by assuming [7], this reasoning presupposes that the correctness of Tarski’s account requires [Co-extensiveness], but I propose to grant this, for the sake of argument.

If this is the argument that Etchemendy has in mind (and I can think of no better), then he is mistaken in thinking that it establishes that [5] and [6] are inconsistent. The most that this argument could demonstrate is that [5], [6], and [7] are inconsistent. And this just reduces to a point we have already granted, namely, that Tarski’s conditions for the logical properties wouldn’t get the right answers in a finite universe, i.e. [5] and [7] are incompatible. But the same is true of Tarski’s account of truth, since it presupposes infinite sequences, infinitely many variables, sentences, sets. Of course, this does not threaten the material adequacy of either account, because in this infinite universe [5] is false.\(^{43}\) Ironically, it looks as though Etchemendy has made the very mistake that he (mistakenly) attributes to Tarski in his discussion of “Tarski’s fallacy” to be discussed in Section 5.\(^{44}\)

4.2. \textit{Conceptual Analysis}

Perhaps it is just too disappointing to understand Tarski’s account as giving a merely material account of the logical properties, because such an account would have no modal force at all. I will say more about
this issue in the next section. Here I wish only to make three brief remarks. First, I think we should look with some suspicion on demands for “modal force” in advance of a clear characterization of the modality at issue.\textsuperscript{45} Without such a characterization, it is not at all clear when the requisite “modal force” is present. After all, it is not implausible that what is wanted is that logical consequences and logical truths be \textit{logically} necessary. But what is logical necessity, and why should we think that it is anything other than what our Tarskian account, say, of logical truth delivers? Second, recall that the material adequacy of the account is not a simple co-extensiveness claim. A definition is adequate just in case it satisfies [Adequacy], and this is not devoid of modal import. Third, while our answer to Etchemendy above committed us to denying that Tarski’s account had the status of a conceptual analysis, the idea that the account was only put forward as a materially adequate one is a \textit{purely historical claim}. Thus, it is still at this point an open question what the modal status of the account can be understood to be. Vann McGee (1992, 1993), Stewart Shapiro (forthcoming), and Gila Sher (1991, 1996) all argue that Tarski’s account can be understood as giving something more than material adequacy, without crediting the account with the status of conceptual analysis. There is a good deal more to be said at this point, as is attested by the fact that it has drawn the attention of these philosophers. This is an active area of study, and I think it remains to be seen just how robust Tarski’s account can be understood to be. However, we will not pursue the matter further here.\textsuperscript{46}

5. ON “TARSKI’S FALLACY”

Etchemendy argues that, in attempting to make a case for his proposed definition, Tarski commits a modal fallacy. Before we can properly discuss this issue, we need to disentangle from Etchemendy’s criticism a confusion which he introduces into the discussion. Etchemendy tells us that if only we could prove

\[ \text{[8]} \] For all \( \varphi \) and \( \Gamma \), \( \varphi \) is a logical consequence of \( \Gamma \) iff for some \( F \), \( \varphi \) is an \( F \)-consequence of \( \Gamma \)

then “Tarski’s definition could hardly be faulted” (p. 86).\textsuperscript{47} He argues that the left-to-right direction of the embedded biconditional

\[ \text{[8L]} \] If \( \varphi \) is a logical consequence of \( \Gamma \), then for some \( F \), \( \varphi \) is an \( F \)-consequence of \( \Gamma \)
should be granted, because of the following argument (cf. pp. 85–86).

Suppose $\varphi$ is a logical consequence of $\Gamma$. This at least implies that if all the sentences of $\Gamma$ are true, then $\varphi$ is true. (Where this conditional is to be construed as material.) Let $F$ be a term function which treats all the terms of the language as logical terms, i.e., maps every term onto itself. It follows that $\varphi$ will be an $F$-consequence of $\Gamma$.

In fact, this argument is invalid, but for the sake of argument, let us grant [8L] as Etchemendy suggests. Then to establish [8], it would only remain to show that the right-to-left direction held.

[8R] If for some $F$, $\varphi$ is an $F$-consequence of $\Gamma$, then $\varphi$ is a logical consequence of $\Gamma$.

But, Etchemendy tells us, [8R] is wildly false. It is painfully obvious that there will generally be choices of $F$ which will make a given $\varphi$ an $F$-consequence of a given $\Gamma$, even though the one is certainly not really a logical consequence of the other. Etchemendy gives the example that 'Lincoln had a beard' is an $F$-consequence of 'Washington was president', for an $F$ which fixes all the terms of these sentences. According to Etchemendy, what makes this situation especially puzzling is that [8R], though so clearly false, seems to follow from a result Tarski claims may be proved.

It can be proved, on the basis of this definition, that every [Tarskian] consequence of true sentences must be true. (Tarski, 1936. p. 417.)

And, though Tarski does not give his proof of this result, Etchemendy sketches a proof on Tarski’s behalf that seems to do the job. In our own terminology, Etchemendy’s reconstructed argument (p. 86) can be put as follows.

Suppose $\varphi$ is a Tarskian consequence of $\Gamma$, and suppose that all of the sentences in $\Gamma$ are true. Let $F$ be a logical term function. There is at least one $d$-sequence, $s$, which is such that (i) for each non-logical term, $\zeta$, in $\varphi$ or $\Gamma$, $s$ assigns to $F(\zeta)$ the actual semantic value of $\zeta$, and (ii) $\text{Dom}(s)$ is the intended domain of discourse for the language. For such an $s$, and for any $\psi$, $s$ satisfies $F[\psi]$ iff $\psi$ is true. Since all the sentences in $\Gamma$ are true, we know that this $s$ satisfies $F[\Gamma]$. But also $\varphi$ is a Tarskian consequence of $\Gamma$, so it follows that $s$ also satisfies $F[\varphi]$. Given the choice of $s$, it now follows that $\varphi$ is true.
So, now we have a genuine paradox: an apparently sound argument for a claim, [CO], which seems to entail an obviously false principle, [8R]. What has gone wrong? On Etchemendy’s diagnosis Tarski has committed a simple modal scope fallacy. If [CO] is construed as

\[\text{[9w]} \quad \text{Necessarily, if for some } F, \varphi \text{ is an } F\text{-consequence of } \Gamma \text{ and every sentence in } \Gamma \text{ is true, then } \varphi \text{ is true}\]

then it is indeed proven by the above argument. However, [9w] does not entail [8R]. The apparent entailment of [8R] by [CO] depends on construing the latter as

\[\text{[9s]} \quad \text{If for some } F, \varphi \text{ is an } F\text{-consequence of } \Gamma, \text{ then necessarily, if every sentence in } \Gamma \text{ is true, then } \varphi \text{ is true.}\]

But [9s] is a stronger claim than [9w], and is not established by the above proof. Thus, Etchemendy charges that a) Tarski mistakenly believed that he had proved something, [CO], which provided evidence of the correctness of his view, because it in turn entailed [8R], but b) what Tarski proved and what entails [8R] differ by the placement of a modal operator. This makes Tarski guilty of the simple modal fallacy of confusing the necessity of the consequence with the necessity of the consequent. This is a rather dramatic revelation, if true, since Tarski was a great logician by any measure. Etchemendy goes on to give two arguments – the argument from bogus consequence and the epistemological argument – which are intended to show that proving only [9w] does not really help establish the correctness of the Tarskian account. This second revelation is of far more than historical interest, because it allegedly removes the main support for the Tarskian/model-theoretic account of the logical properties. Fortunately, Etchemendy’s startling revelation is no true revelation on either count. Again, let me first address the historical issue. A rather revealing presupposition of Etchemendy’s entire discussion of Tarski will thus come to light.

5.1. A Presupposition Uncovered

Tarski does not commit a modal fallacy as charged.\(^{50}\) I will argue that Etchemendy’s argument to the contrary is based on two errors: (i) misunderstanding Tarski’s aim, and (ii) mischaracterizing Tarski’s account. Let’s take a closer look at Etchemendy’s diagnosis. First of all, notice that [8R] contains \textit{no explicitly modal terms at all}. So, why does Etchemendy think that, in order to entail [8R], [CO] must be construed as having an embedded modality (as in [9s])? It is because Etchemendy implicitly assumes [8R] and [9s] to be equivalent, and likewise implicitly assumes
that Tarski would concur. This assumption implies that logical consequence is simply strict implication between two truth claims. We can demonstrate that Etchemendy is making this assumption, and that it plays an essential role in his argument, as follows. Recall that Etchemendy thinks that from

\[[9s]\] If $\varphi$ is a "Tarskian consequence"\(^{51}\) of $\Gamma$, then necessarily, if every sentence in $\Gamma$ is true, then $\varphi$ true

it follows that

\[[8R]\] If $\varphi$ is a "Tarskian consequence" of $\Gamma$, then $\varphi$ is a logical consequence of $\Gamma$

and that it was the desired to establish [8R], and, hence, [8] that tempted Tarski into thinking he had proved [9s]. But what would warrant the inference from [9s] to [8R]? There is surely some implicit premise here. To make this argument deductively valid, we must add the following premise (or something from which it follows).\(^{52}\)

\[[10]\] If it is necessary that if every sentence in $\Gamma$ is true, then $\varphi$ is true, then $\varphi$ is a logical consequence of $\Gamma$.

Which is to say that strict implication is a sufficient condition for logical consequence. Furthermore, Etchemendy explicitly states (p. 82) that it is a necessary condition (as well), i.e.

\[[11]\] If $\varphi$ is a logical consequence of $\Gamma$ then it is necessary that if all the sentences in $\Gamma$ are true then $\varphi$ is true.

I conclude that

\[[12]\] For all $\varphi$ and $\Gamma$, $\varphi$ is a logical consequence of $\Gamma$ iff the truth of all the sentences in $\Gamma$ strictly implies that $\varphi$ is true\(^{53}\)

is an essential presupposition of Etchemendy's "Tarski's Fallacy" criticism, both in the sense that Etchemendy presupposes it to be true, and in the sense that he presupposes that Tarski thought it true.\(^{54}\) But this shows that Etchemendy has misunderstood what notion it was that Tarski was offering an account of. The relation that Tarski wants to capture is a formal relation.

...we are concerned here with the concept of logical, i.e. formal, consequence, and thus with a relation which is uniquely determined by the form of the sentences between which it holds... (Tarski, 1936, p. 414.)
However, strict implication is not a formal relation in this sense. Hence, Tarski’s target notion is not strict implication. The historical attribution that underlies Etchemendy’s argument is surely mistaken. But then, it is hard to see how Tarski could possibly have been tempted into committing a modal fallacy of the sort Etchemendy alleges. So far then, this removes any motivation for construing Tarski’s [CO] as anything other than the unproblematic claim [9w] for which there is a simple proof.

Notice that the mistake which is being attributed to Etchemendy here does not consist in holding that logical consequence is a strict implication between truth claims, though I think this would be mistaken indeed. I guess someone might want to defend such a position, and criticize Tarski’s account for failing to capture all “necessary consequences”, in addition to all the formal ones. But Etchemendy certainly does not do this, instead he develops criticisms that presuppose that Tarski was aiming for something which he evidently was not. We will see more of this in Section 6.

It is also worth pointing out that the assumption that [Co-extensiveness] is an adequacy condition for Tarski’s account also underlies Etchemendy’s whole argument, because it is this presupposition that underlies assumption [8]. In Section 2, I argued that this is mistaken, at least as far as Tarski’s intentions are concerned. For this reason, too, it is hard to see how Tarski could have been tempted toward a modal fallacy, since doing so is supposed to have been in the service of proving something, [Co-extensiveness], which Tarski rejected.

5.2. An Error of Scope

I claimed that Etchemendy’s “Tarski’s Fallacy” criticism also rested on a second mistake, one involving a mischaracterization of Tarski’s account. Etchemendy begins his criticism by remarking that if only one could prove

\[ [8] \text{ For all } \varphi \text{ and } \Gamma, \varphi \text{ is a logical consequence of } \Gamma \text{ iff there is an } F \text{ such that } \varphi \text{ is an } F\text{-consequence of } \Gamma \]

that would certainly establish the correctness of Tarski’s account. Now, according to Etchemendy, it was in trying to prove one half of [8] that Tarski erred by committing a modal scope fallacy. But the real error here is Etchemendy’s, and ironically, it is a quantifier scope error. It is a mistake to think that proving [8] would prove the correctness of Tarski’s account. For that, one would need to prove something at least as strong as

\[ [13] \text{ There is an } F \text{ such that, for all } \varphi \text{ and } \Gamma, \varphi \text{ is a logical consequence of } \Gamma \text{ iff } \varphi \text{ is an } F\text{-consequence of } \Gamma. \]
This is readily apparent from the formulation of Tarski’s account given in Section 2. Let us compare the two parallel clauses, [8R] and the right-to-left clause of [13],

\[ [13R] \text{if } \phi \text{ is an F-consequence of } \Gamma \text{ then } \phi \text{ is a logical consequence of } \Gamma. \]

Immediately, we see that Etchemendy’s reasons for rejecting [8R] do not apply to [13R]. Recall, Etchemendy’s counterexample to [8R] was developed by choosing a funny term function F, so as to get an F-consequence where there was no logical consequence. [13R] is not a complete sentence, and so must be evaluated in its quantificational context, the existential claim [13]. It should be obvious that Etchemendy’s procedure has no force against [13]. The fact that there are some term functions that don’t do the right job goes no way toward showing that there is not one that does.

Though Etchemendy sees that [8R] (and hence [8]) is wildly false – indeed this is crucial to his argument – he fails to take the proper lesson from this, namely that he has mischaracterized Tarski’s account. For it is certain that such a mischaracterization underlies Etchemendy’s belief that proving [8] would establish the correctness of the account. Instead, he goes on to conclude that Tarski must have obscured the obvious falsehood of [8] by committing a modal scope fallacy. We can now see that this charge is not at all justified by Etchemendy’s considerations, because these are vitiated by a quantifier scope error.

5.3. Must Consequences of True Sentences Be True?

We can separate the two mistakes we uncovered in the last two sections off from a number of further issues of historical and philosophical importance. Beyond those errors lies a genuine issue involving modality and the adequacy of Tarski’s account. And it turns out that some of the challenges Etchemendy raises here still threaten even after we unravel the mistakes in his general diagnosis. Tarski does lay claim to [CO] as part of showing that his account is getting the right sort of answers, and that claim has some modal force or other – it should be read either as [9w] or [9s]. Etchemendy argues that there is textual evidence that Tarski thought he had proved [9s]. This challenge remains unmet, because it is quite independent of the troubles we uncovered in the last section. Moreover, Etchemendy also argues on independent grounds that a Tarskian really needs to be able to prove [9s]. This claim is driven by two arguments, one for the conclusion that only proving [9s] provides substantial evidence for the account, and the other to the conclusion that only proving [9s]
shows that the Tarskian account captures truth-preservation in the right sense. So, we have three challenges unmet – one of potential historical, and two of potential theoretical importance. I will discuss each of these in turn. However, since we now know that Etchemendy’s interpretation of this part of Tarski’s text is flawed, it will be well to have available an alternative reading of these key passages in Tarski. Accordingly, I offer my own. Recall that Tarski began his discussion by articulating two intuitive characteristics of consequence.

(a) If a sentence $\varphi$ is a logical consequence of a set of sentences $\Gamma$, then “it can never happen that both the class $\Gamma$ consists only of true sentences and the sentence $\varphi$ is false”. That is, the consequence relation is truth preserving.

(b) A logical or formal consequence relation must be “uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge, and in particular by knowledge of the objects to which the sentence $\varphi$ or the sentences of the class $\Gamma$ refer.” That is, the consequence relation is a formal relation. (pp. 414–415.)

Tarski then combines these characteristics into a precise condition, the Substitution Condition, which he employs in formulating what amounts to a partial adequacy condition, Substitutional Necessity, for any account of logical consequence. After arriving at his proposed definition, Tarski makes a number of claims which are intended to show that his proposed satisfactional account of consequence captures these key features, and so “agrees quite well with common usage.” In particular, Tarski makes the following claims.

**[CO]** It can be proved, on the basis of this definition, that every consequence of true sentences must be true...

**[TH]** In brief, it can be shown that the [Substitution Condition] formulated above is necessary if the sentence $\varphi$ is to follow [in the Tarskian sense] from the sentences of the class $\Gamma$. (p. 417.)

As I understand this passage, Tarski is laying claim to the following results.

**THEOREM [T].** For any $\varphi$ and $\Gamma$, the Substitution Condition is a necessary condition for $\varphi$’s being a Tarskian consequence of $\Gamma$. That is, if $\Gamma \models \varphi$ then, for all respectful replacement functions, $\rho$, if all the sentences in $\rho(\Gamma)$ are true, then the sentence $\rho(\varphi)$ is true. (A detailed proof of this theorem is given in Appendix B.)

**COROLLARY [C].** If $\Gamma \models \varphi$ and if all the sentences in $\Gamma$ are true, then $\varphi$ is true.

It is the provability of the Corollary that grounds Tarski’s claim [CO]. So, I construe [CO] as [9w], where the modal force is logico-deductive
(logical necessity, if you will). Indeed, Etchemendy's reconstructed proof is a proof of this Corollary. Though it will turn out to be important that Etchemendy's reconstruction does not follow Tarski's suggestion in [TH] of proving the result via Theorem T. With this first aperçu of our key passage, let us turn to Etchemendy's three challenges.

5.4. Tarski's Use of Modal Terms

First, Etchemendy claims that Tarski understood [CO] as [9s], and generally thought of his account as having a certain kind of modal force, on the grounds that Tarski uses various modal locutions at key points in his discussion in ways which suggest this. For example, Tarski uses the modal term 'must' in [CO]. My answer to this challenge may already be clear from the interpretation I have offered of Tarski's claims. I think that in every case the modal terms in question can be understood merely as a familiar colloquial use of modal locutions, or as indicating nothing stronger than deductive consequence. Let us take the case of [CO]. I think that Tarski is simply employing the "logician's 'must"' – indicating the logical necessity of the consequence, not the necessity of the consequent. Of course, the necessity of the consequence is borne out by the proof of Corollary C. It is also instructive to note that Tarski elsewhere expresses this same notion of truth-preservation as in [CO] without employing modal terms.\textsuperscript{56} For example:

Formerly it could be assumed that the formalized concept of consequence coincides in extension with that concept in everyday language, or at least that all purely structural operations, which unconditionally lead from true statements to true statements, could be reduced without exception to the rules of inference employed in the deductive sciences. (Tarski, 1933, p. 294.)

Likewise, Tarski's use of the term 'necessary' in [TH] indicates only that Substitutional Necessity is a necessary condition for Tarskian consequence. Compare Tarski's words in [TH], with an earlier passage, [EP], where he remarks that the Substitution Condition is necessary but not sufficient for logical consequence:

[EP] In the [Substitution Condition] we have obtained a necessary condition for the sentence \( \varphi \) to be a consequence of the class \( \Gamma \). The question now arises whether this condition is also sufficient... (p. 415.)

In brief, it can be shown that the [Substitution Condition] formulated above is necessary if the sentence \( \varphi \) is to follow [in Tarski's defined sense] from the sentences of class \( \Gamma \). On the other hand, this condition is in general not sufficient... (p. 417.)
I think this first challenge of Etchemendy’s is thus easily met. *We will henceforth assume that (so far as has been shown) Tarski’s account entails [9w] but not [9s], and that Tarski’s [CO] does not claim otherwise.* So, we may say that Tarski’s claim should be construed weakly.

5.5. *The Argument from Bogus Consequence*

Etchemendy’s second and third challenges are of rather more philosophical interest, because he purports to show that Tarski’s results (weakly construed) are so weak as to provide no assurance at all for thinking that the Tarskian/model-theoretic account is even extensionally correct. In particular, Etchemendy claims that Corollary C “follows trivially from the definition, but [that this] hardly seems much evidence that [Tarskian consequence] ‘agrees quite well with ordinary usage’” (p. 90). Etchemendy supports his claim in two ways. First, he gives an example of an obviously very faulty definition of consequence for which a suitable version of the Corollary is said to hold (p. 90). Second, Etchemendy argues that if Tarski’s account fails to entail something like [9s], it fails to capture the important epistemological sense in which logical consequences are “guaranteed to preserve-truth” (p. 93). I will treat the second argument in the next section. Here, I will show that Etchemendy’s first argument is flawed.

**DEFINITION [bogus].** \( \varphi \) is a *bogus* consequence of \( \Gamma \) if \( \varphi \) is true or some member of \( \Gamma \) is false.

**COROLLARY [C1].** If \( \varphi \) is a *bogus* logical consequence of \( \Gamma \) and if all the sentences in \( \Gamma \) are true, then \( \varphi \) is true.

Etchemendy argues that proving Corollary [C] for Tarskian consequence shows nothing, because we can do likewise for bogus consequence, and the latter is obviously unsatisfactory. However, Etchemendy’s example is subject to an easy complaint. The definition allows us to derive a suitable correlate of Corollary C, *but not of Theorem T*. But Substitution Condition \(_1^1\) is a necessary condition for logical consequence, as Tarski was careful to point out, and so, Substitutional Necessity is a condition of adequacy for any account of logical consequence. It is in the very passage that we are concerned with that Tarski claims that his account satisfies this condition. However, this central claim comes in a portion of the passage that Etchemendy has overlooked, namely [TH]. As we saw, Etchemendy proves Corollary C in a way that wholly bypasses Theorem T.
These considerations have immediate ramifications for Etchemendy's example. As already noted, we can prove a version of the Corollary for Etchemendy's bogus_{1} logical consequence, but we cannot prove a corresponding version of Theorem T. In other words, bogus_{1} logical consequence and Tarskian consequence differ in this respect: the former fails to meet a Tarskian condition of adequacy for accounts of logical consequence, but the latter succeeds. Etchemendy's obviously faulty definition violates Substitutional Necessity, which, by his own lights, is a central principle of Tarski's account.

A closer look at that key passage in Tarski reveals that bogus_{1} consequence falls even shorter of the mark even than this, and at the same makes it clear that coming up with an example of the sort Etchemendy needs for this critical argument is not going to be easy. Tarski says in that crucial passage that

[CO] It can be proved, on the basis of this definition, that every consequence of true sentences must be true,

and also that the consequence relation which holds between given sentences is completely independent of the sense of the extra-logical constants which occur in these statements.

[FM] In brief, it can be shown that the [Substitution Condition] formulated above is necessary if the sentence $\varphi$ is to follow [in the Tarskian sense] from the sentences of the class $\Gamma$.

[TH] On the other hand, this condition [Substitution Condition] is in general not sufficient, since the concept of consequence here defined (in agreement with the standpoint we have taken) is independent of the richness in concepts of the language being investigated. (p. 417)

Rightly understood, I think this passage may be glossed as the claim that 'Tarskian consequence' satisfies the following.

(1) X is provably truth-preserving.
(2) X is provably a formal-logical relation.
(3) We can prove (1) is satisfied by X by first proving that [Substitution Condition] is a necessary condition for X, if $\varphi$ is to follow (in the sense of X) from $\Gamma$.
(4) [Substitution Condition] is not a sufficient condition for X.
(5) X is independent of the richness in concepts of the language being investigated.

As I count them, there are five different claims made by Tarski on behalf of his account. Etchemendy's bogus_{1} consequence only satisfies (1). But, of course, Tarskian consequence satisfies all of (1)–(5). We have
already discussed (1) and (3). (3) is established by proving Theorem T in Appendix B, and [C] is an easy corollary, which in turn establishes (1). Claim (2) is established by a proof of [FM] in Appendix C. The fact that (4) holds can be demonstrated easily by counterexample, e.g. using the ‘heart’-‘kidneys’ language we considered earlier. Claim (5), in so far as it says something in excess of claim (2), would have to be established by suitably generalizing the proof given in Appendix C. However, there is a concern about how (5) can be formulated in a way compatible with Tarski’s way of avoiding the semantic paradoxes. Thus, this last, while evidently intuitively true is not said to be formally provable.

The weakness of Etchemendy’s argument should now be clear. Of course, material consequence is truth-preserving, and if this was all we could claim on behalf of Tarski’s account of logical consequence, that would be problematic. However, Etchemendy is mistaken in thinking that is all that can be, or was, claimed for Tarski’s account. The fact that it is not hard to give an obviously wrong account that satisfies any one, or even two, of (1)–(5) proves nothing against Tarski’s account. The real challenge for Etchemendy is to produce an obviously wrong account that satisfies all of (1)–(5). I have suggested that (1)–(5) show that Tarskian consequence satisfies a genuine adequacy condition, namely, [Adequacy]. Given this, it is hard to see how any definition that satisfied (1)–(5) could be obviously wrong. Well, a Tarskian can afford to wait.

Let’s take stock. Properly understood, Tarski’s account entails [9w], not [9s], and we have argued that Tarski thought no different. Etchemendy’s argument from bogus consequence was designed to show that, so understood, Tarski’s account was inadequate. However, this argument has been undermined. Its apparent force relied on ignoring the several results that Tarski is able to prove on behalf of his account.

5.6. The Epistemological Argument

Still, Etchemendy has a second argument to show that Tarski’s account is inadequate, insofar as it does not entail [9s]. The claim here is that the account fails to capture the important sense in which logical consequences are “guaranteed to preserve truth” (p. 93). Again, this charge, if true, would apply mutatis mutandis to the model-theoretic account. Etchemendy characterizes the sense in which Tarski’s account fails to correctly capture the truth-preservational aspect of logical consequence in the following way:

It must be possible to come to know that the conclusion [of an argument] is true on the basis of knowledge that the argument is valid and that its premises are true. This is a feature of logically valid arguments that even the most skeptical of modal notions recognize.
as essential... [Regardless of the choice of fixed terms, Tarski’s account] still misses [this] essential feature of validity... For in general, it will be impossible to know whether an argument [meets the conditions for Tarskian consequence] without antecedently knowing the specific truth values of its constituent sentences... Tarski’s fallacy obscures this omission, by noting that arguments declared valid are indeed guaranteed to preserve truth. But this is not the required guarantee: it is backed up only by the definition of validity, not by any characteristic of the argument itself, whether modal, epistemic, or semantic. Consequently, it leaves such arguments impotent as a means of justifying their conclusions. (Etchemendy, 1990, p. 93.)

This is a very puzzling passage, because of course, the conditional epistemic warrant that Etchemendy claims is not vouchsafed by Tarski’s account is clearly secured by Corollary C. That is, if you had “knowledge that the argument is valid and that its premises are true”, then since Corollary C is derivable, you would certainly be in a position to know that the conclusion of the argument is true. Isn’t this the guarantee Etchemendy asks for at the beginning of the quoted passage? As regards that guarantee being “backed up only by the definition of validity, not by any characteristic of the argument itself”, evidently there is some confusion here. I should have thought that the definition of validity (aka Tarskian consequence) was given in terms of “characteristics of the argument itself”. If the components of an argument stand in the Tarskian-consequence relation, that always reflects some special characteristics of the argument itself, surely.58

In summary, I have argued that Tarski was not attempting to prove the strong claim [9s] that Etchemendy alleges, and hence no fallacy was committed. Etchemendy also maintains that an account which only nets the weaker claim is inadequate on two fronts. I have shown that Etchemendy’s justification for this claim is seriously wanting on both counts. Still, some philosophers have acceded to Etchemendy’s demand for the stronger claim, and tried to defend Tarski in a very different manner. Recall that the debate about “Tarski’s Fallacy” has centered around Etchemendy’s reconstruction of Tarski’s argument for [CO]. Thus, there is another way of responding to Etchemendy’s criticism, which has been taken up by both Vann McGee (1992, 1993), and Gila Sher (1991, 1996). Both seek alternative arguments to offer on Tarski’s behalf which entail the strong modal conclusion, [9s].59 But I have argued that it is a mistake to think Tarski was arguing for [9s], and I have undermined all of Etchemendy’s arguments which seemed to show such an argument was needed in any case. So, I think the shoe is still on the other foot, it still remains for the opponent of Tarski’s account to give a clear, strong argument to show that the account is inadequate unless it yields [9s].
6. ON THE POSSIBILITY OF A PRIVILEGED CLASS OF LOGICAL TERMS

I have defended Tarski's account against each of the criticisms we have discussed, and affected a complete reconciliation between that account and the model-theoretic account. The final criticism we will consider is one that, if true, would show that the existential claim, [Existence], crucial to Tarski's account (and so, by extension, to the model-theoretic account) is false. Recall that the existential claim is just the claim that there are logical term functions for all the languages in some broad class of languages \( \mathcal{K} \). Etchemendy argues that there are very simple (even first-order) languages for which the distinction between logical and non-logical terms cannot be drawn successfully. That is not just to say that we are not sure how to draw the distinction in such languages, but that no division of terms will make Tarskian consequences in the language correspond intuitively to logical consequences. This is a startling claim. It should also be clear that restricting \( \mathcal{K} \) to avoid Etchemendy's counterexample languages is not a live option, because clearly the resulting account would be insufficiently general, being inapplicable even to some first-order languages. Etchemendy uses his result to argue further that it is a mistake to think that various problems he has raised for Tarski's account could somehow be avoided by correctly identifying some privileged class of logical terms. He holds that philosophers and logicians who have worked toward identifying such a class have been pursuing a chimera.\(^6\)

6.1. The "Bullet" Argument

Let us take a closer look at the argument Etchemendy offers for his claim. First, we will need two closely-related languages with which to work. Following Etchemendy,

let \( L \) be a very simple language with two names ('George Washington' and 'Abe Lincoln'), two predicates ('was president' and 'has a beard'), three truth-functional operators ('\( \lor \)', '\( \land \)' and '¬'), where all these terms have their usual meaning.

Let \( L^* \) be a language just like \( L \), except there is, in addition, a binary connective, '●', with the following truth condition

for any sentences \( \varphi \) and \( \psi \) of \( L^* \), \( r(\varphi \bullet \psi) \) is true iff Lincoln had a beard and either \( \varphi \) is true or \( \psi \) is true.

Etchemendy makes his case, using these languages, in the following way.
[L* is] a language in which every sentence is logically equivalent to some sentence of [L]. Yet it is easy to show (and not surprising in the least) that no selection of F yields a set of "logical truths" containing exactly the right sentences – that is, those equivalent to logical truths of the original language. If we include ‘•’ in F, the account overgenerates; if we exclude it, the account undergenerates. (Etchemendy, 1990, p. 134.)

I will show that this argument is not sound. For definiteness, let us restrict our attention to two candidates for the logical terms of L*. Let F be such that the only F-terms of L* are ‘¬’, ‘∧’ and ‘∨’. Let F* be such that the only F*-terms of L* are ‘¬’, ‘∧’, ‘∨’, and ‘•’. Assume, as is reasonable, that the logical terms of L are just ‘¬’, ‘∧’ and ‘∨’. What are the logical terms of L*? Presumably, all the logical terms of L are also logical terms of L*, since their meanings do not differ between the two languages. So, the logical terms of L* must be either the F-terms or the F*-terms of L*.

It all comes down to whether ‘•’ is to be counted as a logical term or not. I gather that this is the sort of set-up that Etchemendy has in mind for the argument he alludes to.

According to Etchemendy, if we take the logical terms of L* to be the F*-terms and so count ‘•’ as logical, some sentences which are not genuinely logical truths will nonetheless be F*-true, and so satisfy Tarski’s conditions for logical truth. For example, any sentence of L* of the form \( (\varphi \bullet (\neg \varphi)) \) would be declared a Tarskian logical truth, and this certainly seems wrong, since that sentence entails that Lincoln had a beard. From this we can conclude that ‘•’ may not be treated as a logical term. This result is not by itself worrisome, however, since, ‘•’ is not a likely candidate for a logical term anyway.

The trouble with Etchemendy’s argument surfaces when we spell out its second half. The claim is that, if we take the logical terms of L* to be the F-terms, some sentences which are genuinely logical truths of L* will not be F-true, and so will not satisfy Tarski’s conditions for logical truth. Etchemendy thinks that there must be such logical truths since there will be sentences of L* which are not F-true, but which are nonetheless logically equivalent to sentences of L which are themselves logical truths of L. Evidently, then, Etchemendy’s argument relies crucially on the following assumption.

If \( \varphi^* \) in L* is logically equivalent to \( \psi \) in L and \( \psi \) is a logical truth of L, then \( \varphi^* \) is a logical truth of L*. Or, as we might put it, sentences logically equivalent to logical truths are themselves logical truths.

But what notion of “logical equivalence” is being invoked here? Though Etchemendy does not spell out his argument in detail, I will hazard a
guess as to how the full reasoning is supposed to go. Let ‘B’ be short for ‘Lincoln had a beard’. Let \( \varphi \) be any sentence common to \( L \) and \( L^* \).

1a. We can derive from the semantic stipulations of \( L^* \) that 
\[ \neg (\varphi \land \neg \varphi) \uparrow \text{is true (in } L^* \text{)} \iff \text{Lincoln had a beard and either } \varphi \text{ is true (in } L^* \text{) or } \varphi \text{ is not true (in } L^* \text{).} \]

1b. We can likewise derive from the semantic stipulations of \( L \) that 
\[ \neg (B \land (\varphi \lor \neg \varphi)) \uparrow \text{is true (in } L \text{)} \iff \text{Lincoln had a beard and either } \varphi \text{ is true (in } L^* \text{) or } \varphi \text{ is not true (in } L^* \text{).} \]

2. It follows easily that 
\[ \neg B \lor (\varphi \land \neg \varphi) \uparrow \text{is true (in } L^* \text{) iff } \neg B \lor (B \land (\varphi \lor \neg \varphi)) \uparrow \text{is true (in } L \text{)} \text{. So, these are “logically equivalent” sentences.} \]

3. But 
\[ \neg B \lor (B \land (\varphi \lor \neg \varphi)) \uparrow \text{is certainly a logical truth of } L. \]

X. Sentences logically equivalent to logical truths are themselves logical truths.

5. From [2], [3], and [X], it follows that 
\[ \neg B \lor (\varphi \land \neg \varphi) \uparrow \text{is a logical truth of } L^*. \]

6. However, 
\[ \neg B \lor (\varphi \land \neg \varphi) \uparrow \text{is not an F-truth of } L^*. \text{ Thus, using } F, \text{Tarski’s condition fails to recognize at least one logical truth of } L^*. \]

The trouble with this argument is the notion of “logical equivalence” that must be invoked to make the argument valid. However plausible principle [X] may seem, there is good reason to think that it is false, given the sense of “logically equivalent” being employed. In order for Etchemendy to make good on his argument, he needs something at least as strong as the following to hold.

\[ \text{[EQ] For sentences } \varphi \text{ in } L' \text{ and } \psi \text{ in } L'', \text{ if it follows from the satisfaction (or truth) conditions of } L' \text{ and } L'' \text{ that } \varphi \text{ is true (in } L' \text{) if and only if } \psi \text{ is true (in } L'' \text{), then } \varphi \text{ and } \psi \text{ are “logically equivalent”.} \]

The validity of Etchemendy’s argument depends on some such condition being true. The argument needs to establish that a “logical-status-preserving” relation obtains between 
\[ \neg B \lor (\varphi \land \neg \varphi) \uparrow \text{ and the logical truth } \neg B \lor (B \land (\varphi \lor \neg \varphi)) \uparrow \text{. Etchemendy calls this relation “logical} \]
equivalence”, but *whatever we call it*, it is a relation that must be determined to hold between these two sentences in virtue of assumptions that have been made by the argument up through step (2). [EQ] merely codifies this. [EQ] says, in essence, that two sentences that meet the going assumptions of our two target sentences bear the specified relation to each other, and [X] says that the specified relation is logical-status preserving. It should be obvious that both these claims are essential to the argument. However, we can reduce the joint assumption of [EQ] and [X] to absurdity as follows.62

1. Suppose that the language $L^+$ has the predicates, ‘is unmarried’, and ‘is a male’, where these have their usual meaning. Our language will have one other predicate, ‘is a bachelor’. And we will outfit the language otherwise with just the classical logical operators.

2. Suppose that ‘is a bachelor’ has the following satisfaction conditions in $L^+$. A thing satisfies ‘$x$ is a bachelor’ iff that thing is both unmarried and a male.

3. Applying [EQ], it will follow that ‘all bachelors are unmarried’ is “logically equivalent” to ‘all unmarried males are unmarried’.

4. ‘all unmarried males are unmarried’ is a logical truth (in the target pre-theoretic sense, as well as the Tarskian and model-theoretic senses).

5. Applying principle [X], we conclude ‘all bachelors are unmarried’ is a logical truth.

But, of course, ‘all bachelors are unmarried’ is *not* a logical truth (in the target pre-theoretic sense, or in either the Tarskian or model-theoretic senses), so I take that for a *reductio*. Put somewhat differently, the above argument reveals that Etchemendy’s bullet argument presupposes that necessary (or at least analytic) truths are logical truths, but this is just another manifestation of a misunderstanding of the target of Tarski’s account. Tarski’s target notions are formal-logical notions, i.e., notions that are “completely independent of the sense of the extra-logical terms”. Necessary truth and analytic truth are not formal logical notions in this sense, just as strict implication is not. I cannot see how Etchemendy’s argument could run without employing this false presupposition, so I conclude that Etchemendy’s “bullet language” argument against Tarski’s account is hopelessly flawed.
There is yet room for Etchemendy to object to this line of argument. He might claim for true, what his argument presupposes, that necessary (or at least analytic) truths, like the one about bachelors, are logical truths. While apparently Etchemendy does think this,63 I don’t think that he can appeal to his own unorthodox view to rescue the current argument, on pain of his completely missing the target of Tarski’s account. Tarski’s target notions are formal-logical notions. The necessary truth of the bachelor sentence is just not a formal-logical matter.

This is not to say that Etchemendy cannot object to Tarski’s account on the grounds that it does not treat analytic truths as a (perhaps improper) subclass of logical truths. However, that objection requires no argument to register and substantive argument to justify. But this cause is not at all furthered by the sort of argument that Etchemendy gives. Tarski explains early in his 1936 essay that the notions he is interested in are formal-logical notions. He takes such formality as a condition of adequacy for any account of these notions. Etchemendy can complain about this characterization, but neither his complaint, nor his case against Tarski, can be strengthened by mounting an argument against Tarski that simply applies this underlying disagreement. In particular, the resulting argument could not be telling us anything about the existence of logical term functions, nor about whether Tarski succeeded in capturing the notions he sought to capture, nor, for that matter, whether the model-theoretic account succeeds in capturing those notions.

7. CONCLUSION

I have considered four serious criticisms which Etchemendy raises against Tarski’s account of the logical properties, and I have shown that each of these criticism is just as seriously mistaken. I have focused on these particular criticisms because it seemed to me that, in answering to them, (1) a clearer overall picture would emerge of (i) Tarski’s own account, and (ii) Etchemendy’s troubled analysis, and (2) a sustained defense of Tarski’s account would be achieved. Regarding the first of these (1ii), in the course of this essay I hope to have crafted a precise and faithful rendition of Tarski’s account and, at every turn, given the most plausible understanding of his aims and the claims he makes for his account that can be drawn from a careful reading of the texts.

Regarding the second, (1ii), we uncovered a number of rather important failings in Etchemendy’s critical analysis. Not to put too fine a point on it, these included (i) a misunderstanding of Tarski’s aims, both in terms of subject and conceptual status, (ii) a mischaracterization of
Tarski’s account, and (iii) inattention to several important claims Tarski makes on behalf of his account.

Regarding the third, (2), I have argued that, after all, Tarski’s 1936 account offers an evidently materially adequate account of the logical properties that is, essentially, the model-theoretic account so familiar today. None of Etchemendy’s arguments to the contrary withstood careful scrutiny. So, in particular, they do not at all show that Tarski’s account is significantly different or incompatible with the model-theoretic account, nor that these accounts admit of simple counterexamples, nor that the accounts are supported by a specious argument in which Tarski commits a simple modal fallacy, nor that the distinction between logical and non-logical terms is bankrupt. As far as I can see, nothing remains of Etchemendy’s arguments in the face of the considerations I have brought forward. I take that to constitute a defense of Tarski. Of course, there are genuine philosophical issues to be raised about logical consequence and about Tarski’s account, and I have not attempted here to say much by way of positive remarks on these issues. But, first line of defense first. The rest must wait.

APPENDIX A

Unrelativized Sequences Are Problematic

If there is no provision in Tarskian sequences for specification of a domain of discourse (or some other means of affecting domain relativization), then quantifiers cannot be treated as logical terms.

Let \( F \) treat the following symbols as logical terms: ‘\( \exists \)’, ‘=’, ‘¬’. 64

‘\( \exists x \exists y \ x \neq y \)’ is logically true iff every sequence \( s \) satisfies \( F[\exists x \exists y \ x \neq y] \). But, under current assumptions \( F[\exists x \exists y \ x \neq y] = \\exists x \exists y \ x \neq y \). So, ‘\( \exists x \exists y \ x \neq y \)’ is logically true iff every sequence \( s \) satisfies ‘\( \exists x \exists y \ x \neq y \)’. Let \( s \) be an arbitrary sequence.

\( s \) satisfies ‘\( \exists x \exists y \ x \neq y \)’ iff there is \( s_1 \) such that \( s_1 \) is an ‘\( x \)’-variant of \( s \) and \( s_1 \) satisfies ‘\( \exists y \ x \neq y \)’.

iff there are \( s_1 \) and \( s_2 \) such that \( s_2 \) is a ‘\( y \)’-variant of \( s_1 \), and \( s_1 \) is an ‘\( x \)’-variant of \( s \) and \( s_2 \) satisfies ‘\( x \neq y \)’.
Of course, \( s_2 \) satisfies \( \mathrm{iff} \) \( s_2(f('x')) \) is not identical to \( 'x \neq y' \), \( s_2(f('y')) \).

It remains only to show that for any given sequence \( s \), there exist sequences \( s_1 \) and \( s_2 \) of the appropriate sort. But, of course, there are such sequences. Just let \( s_2 \) be like \( s \) except let two distinct things be in the \( 'x' \)th and \( 'y' \)th positions. If you like, we could even put the symbols \( 'x' \) and \( 'y' \) themselves into the \( 'x' \)th and \( 'y' \)th positions of \( s_2 \). By this, we have shown that every \( s \) satisfies \( \exists x \exists y \; x \neq y \), and so \( \exists x \exists y \; x \neq y \) comes out as a logical truth on this choice of fixed terms.

But it is evident that \( \exists x \exists y \; x \neq y \) is not a logical truth.

APPENDIX B

Proofs for Theorem T & Corollary C

I will first give the argument in a context without domain relativization, and then remark how the argument can be modified so as to take domain relativization into account. We will employ the following definitional statements about truth and Tarskian logical consequence.

DEFINITION [For any sentence \( \varphi \), set of sentences \( \Gamma \), and where \( 's' \) ranges over sequences].

(A) \( \varphi \) is true \( \mathrm{iff} \) \( \forall s, \; s \models \varphi \).
(B) \( \Gamma \models \varphi \) \( \mathrm{iff} \) there is a logical term function \( F \) for \( L \) such that, \( \forall s \), if \( s \models F[\Gamma] \) then \( s \models F[\varphi] \).

In addition, we will make use of the following propositions.

LEMMA [For any logical term function, \( F \), and respectful replacement function \( \rho \), and where \( 's' \) ranges over sequences, and \( '\varphi' \) ranges over sentences].

(1) \( \exists s \; s \models \varphi \; \mathrm{iff} \; \forall s \; s \models \varphi \).
(2) \( \exists s \forall \rho \; (s_1 \models \varphi \; \mathrm{iff} \; s_1 \models F[\varphi]) \).
(3) \( \forall s \exists s_2 \forall \varphi \; (s \models F[\rho(\varphi)] \; \mathrm{iff} \; s_2 \models F[\varphi]) \).

The first of these lemmas is a simple fact about the Tarskian notion of satisfaction as it pertains to (closed) sentences, and is proved in (Tarski,
1933). To see that Lemma 2 is also true, it will be enough to give a precise characterization of the sort of sequence that it describes. To get an $s_1$ that works, we want to ensure that the extra-linguistic variables get assigned by $s_1$ to the actual extensions of the terms which those extra-linguistic variables replace under $F$. So, for each non-logical term, $\zeta$, we will let $s_1(f(F[\zeta])) = \text{the actual extension of the term } \zeta$. For Lemma 3 it will also be enough to give a precise characterization of the sort of sequence, $s_2$, which would be required for a given sequence, $s$. In this case, for each non-logical term, $\zeta$, we will let $s_2(f(F[\zeta])) = s(f(F[\rho(\zeta)]))$.

**THEOREM [T].** If $\Gamma \models_T \varphi$ then the Substitution Condition holds for $\Gamma$ and $\varphi$. That is, if $\Gamma \models_T \varphi$ then for all respectful replacement functions $\rho$, if all the sentences in $\rho(\Gamma)$ are true, then $\rho(\varphi)$ is true.

**Proof.** Suppose that $\Gamma \models_T \varphi$. So, by definition, there is a logical term function, $F$, for $L$ such that, for every sequence $s$, if $s \models F[\Gamma]$ then $s \models F[\varphi]$. Let $F$ be just such a function. Let $\rho$ be any respectful replacement function for $L$, and suppose that all the sentences in $\rho(\Gamma)$ are true. So, by the definition of truth, $\forall s, s \models \rho(\Gamma)$.

By Lemma 2, $\exists s_1 \forall \psi (s_1 \models \psi \text{ iff } s_1 \models F[\psi])$. Let $s_1$ be just such a sequence. Now, $s_1 \models \rho(\Gamma)$, since every sequence does. By choice of $s_1$, it follows that $s_1 \models F[\rho(\Gamma)]$. By Lemma 3, $\exists s_2 \forall \psi (s_1 \models F(\rho(\psi)) \text{ iff } s_2 \models F[\psi])$. Let $s_2$ be just such a sequence. Thus, $s_2 \models F[\Gamma]$. But then, by hypothesis, it follows that $s_2 \models F[\varphi]$. By choice of $s_2$, we can then infer that $s_1 \models F[\rho(\varphi)]$, and by choice of $s_1$, infer from this that $s_1 \models \rho(\varphi)$. By Lemma 1, we conclude that $\forall s, s \models \rho(\varphi)$. Applying the definition of truth again, we conclude that $\rho(\varphi)$ is true, which was to be shown.

Thus, given $\Gamma \models_T \varphi$, we have argued that, for any $\rho$, if $\rho(\Gamma)$ is true, then $\rho(\varphi)$ is true, i.e. that the Substitution Condition holds for $\Gamma$ and $\varphi$. □

The above argument took no special account of the domain relativization which it is necessary to assume in order to understand Tarski's account as fundamentally in line with the modern model-theoretic account. In particular, if sequences themselves specify a domain with respect to which quantifiers are evaluated (as determined by their satisfaction clauses in a Tarskian truth theory), then Lemma 1 will be false. So, it is important to see that the argument we have constructed goes through with domain relativization on board.

Following Tarski (1933, pp. 199–207), let us introduce the requisite domain relativization by taking satisfaction to be a three-place relation – between a sequence, a formula, and a domain of individuals – along
the lines suggested in Section 3.1. For a given domain of individuals, $D$, let $\text{SeqD}$ be the class of sequences of extensions that can be built set-theoretically from $D$. Tarski gives the following definition. A sentence $\varphi$ is true (in $L$) relative to an individual domain, $D$, iff for all $s \in \text{SeqD}$, $s$ satisfies $\varphi$ relative to individual domain $D$. Then, according to Tarski we can define absolute truth as follows. A sentence $\varphi$ is true (in $L$) relative to individual domain $D^*$, where $D^*$ is the domain that includes all individuals (in the intended domain of discourse for $L$). Our definitions and lemmas can now be systematically relativized and generalized as follows.

**DEFINITION** [For any sentence $\varphi$, set of sentences $\Gamma$, and where 's' ranges over sequences].

(A) $\varphi$ is true iff $\forall s \in \text{SeqD}^*$, $s \models_D \varphi$.

(B) $\Gamma \models_T \varphi$ iff there is a logical term function $F$ for $L$ such that, $\forall D \forall s \in \text{SeqD}$, if $s \models_D F[\Gamma]$ then $s \models_D F[\varphi]$.

**LEMMAS** [For any logical term function, $F$, and respectful replacement function $\rho$, and where 's' ranges over sequences, and 'varphi' ranges over sentences].

(1) $\forall D \exists s \in \text{SeqD} \ s \models_D \varphi$ iff $\forall s \in \text{SeqD} \ s \models_D \varphi$.

(2) $\exists s_1 \in \text{SeqD}^* \ \forall \varphi \ (s_1 \models_{D^*} \varphi \text{ iff } s_1 \models_{D^*} F[\varphi])$.

(3) $\forall D \forall s \in \text{SeqD} \ \exists s_2 \in \text{SeqD} \ \forall \varphi \ (s \models_D F[\rho(\varphi)] \text{ iff } s_2 \models_D F[\varphi])$.

For the body of the proof, we will, in similar fashion, relativize all quantifications over sequences to quantifications over the sequences of $\text{SeqD}^*$, and correspondingly relativize all satisfaction relations to $D^*$. The result will be a valid argument which incorporates an explicitly domain-relativized version of Tarskian consequence.

**COROLLARY [C].** If $\Gamma \models_T \varphi$ then, if all the sentences in $\Gamma$ are true, then $\varphi$ is true.

*Proof.* Let $\rho$ be the trivial respectful replacement function for $L$, i.e. the identity function on the non-logical terms of $L$. Then apply Theorem T.

\[\square\]

**APPENDIX C**

*Proof for [FM]*

We wish to prove that the Tarskian consequence relation between statements is "completely independent of the sense of the extra-logical con-
stants which occur in these statements". Of course, the result is rather obvious, because of the way Tarskian consequence is defined in terms of satisfaction. But what can we do by way of demonstration? It seems there are various approaches we might use (and Tarski does not give any hint about his own approach). Here is one. First, let's introduce some terminology.

DEFINITION. Two languages, $L$ and $L^*$ are $F$-alike if, $L$ and $L^*$ have the same vocabulary, assign to each term the same semantic category, have the same formation rules, have the same domain(s) of quantification, $F$ is a logical term function for both $L$ and $L^*$, and all of the $F$-terms have the same meaning in $L$ and $L^*$.66

Any language that is $F$-alike a given language, $L$, differs from $L$ at most in "the senses of the extra-logical constants". Thus, the following theorem shows that Tarskian consequence for a given language $L$ is independent of the senses of $L$'s non-logical terms, by showing that Tarskian consequence is invariant between $F$-alike languages.

THEOREM [F]. If $F$ is a logical term function for $L$, then for all languages $L^*$ that are $F$-alike $L$, for all $\varphi$ and $\Gamma$ in $L$ (and hence also in $L^*$),

$$[14] \quad \varphi \text{ is a Tarskian consequence of } \Gamma \text{ in } L \text{ iff } \varphi \text{ is a Tarskian consequence of } \Gamma \text{ in } L^*.$$

Proof. Since, $L$ and $L^*$ are $F$-alike, $F$ is a logical term function for $L^*$ as well as $L$. So, $F$-consequences in $L$ and $L^*$ are Tarskian consequences in $L$ and $L^*$, respectively. Now, since $L$ and $L^*$ have a common pool of sentences, and have a common term function $F$, it follows that the substitution function $F[\ ]$, since it is a purely syntactic operation, is independent of any differences between $L$ and $L^*$. That is, $F[\psi]$ gives us the same formula regardless of whether $\psi$ is considered a sentence of $L$ or $L^*$. But then, the condition,

$$\text{for all } d\text{-sequences, } s, \text{ if } s \text{ satisfies } F[\Gamma] \text{ then } s \text{ satisfies } F[\varphi],$$

gives a necessary and sufficient condition for $\varphi$'s being an $\Gamma$-consequence of $\Gamma$ in $L$ as well as giving a necessary and sufficient condition for $\varphi$'s being an $F$-consequence of $\Gamma$ in $L^*$. Since, by choice of $F$, $F$-consequence for these languages is Tarskian consequence, [14] follows immediately. □

In this way we may prove that Tarskian consequence is independent of the senses of non-logical terms of a language. Again, the result is
unsurprising, since whether $\varphi$ and $\Gamma$ are Tarskian-consequence related depends only on the relation which obtains between the formula $F[\varphi]$ and the set of formulas $F[\Gamma]$, and these are formulas with no non-logical terms in them.

At one point, Tarski claims that "the concept of consequence here defined (in agreement with the standpoint we have taken) is independent of the richness in concepts of the language being investigated" (p. 417). Possibly, this claim was meant to be no more than a restatement of the above claim that logical consequence is "independent of the sense of the extra-logical constants". However, it sounds to me like a stronger claim, and I have treated it as such throughout this paper. It seems to me that, in order to establish this claim, one would have to prove a result similar to Theorem F, but where the notion of F-alike was suitably expanded (call the new relation, 'F-alike**) so as to include pairs of languages, L and $L^*$, where $L^*$ was an extension of L which included additional non-logical vocabulary, and perhaps also different domain(s) of quantification.

**PROPOSITION [F**]. If $F$ is a logical term function for L, then for all languages $L^*$ that are F-alike* L, for all $\varphi$ and $\Gamma$ in L (and hence $L^*$), [$\varphi$ is a Tarskian consequence of $\Gamma$ in L iff $\varphi$ is a Tarskian consequence of $\Gamma$ in $L^*$].

As discussed in Section 2, the trouble for Tarski with this proposition would have been that it quantifies over all extensions of the target language L, and this forces the language in which the proposition is expressed out of the Tarskian hierarchy. For this reason, [Adequacy] must remain an informal condition in Tarski's account, if, indeed, (F*) is needed over and above (F) in order to establish that Tarskian consequence is formal-logical.

**NOTES**

1 I take the model-theoretic account to be an account according to which a sentence $\varphi$ of L is a logical consequence of a set of sentences $\Gamma$ of L just in case every L-model in which all the sentences in $\Gamma$ are true, is also a model in which $\varphi$ is true. Where an L-model is a set-theoretical structure that specifies a domain of discourse, and assigns extensions over the domain of discourse to the constants and predicates of L, and where truth-in-a-model is defined in the usual way. This characterization must be suitably generalized for higher-order languages.

2 Page references in this essay are to (Tarski, 1936) or (Fitchemendy, 1990), unless otherwise indicated.

3 Set S is freely generated from a base set of symbols B by a collection of concatenation operations C. just in case S is the least set that contains all the expressions that can be built up from the base set by a finite number of applications of the concatenation operations.
\(\tau\) can be said to partition \(S\) just in case \(\tau\) maps each symbol in \(S\) to one and only one semantic category. Formally, we can simply require that \(\tau\) be a total function from \(S\) into, say, the set of natural numbers. Tarski (1933, p. 216) gives a rough syntactic characterization of the notion of a semantic category: two terms are in the same semantic category just in case intersubstitution of them preserves well-formedness. However, we will eventually need all terms in any given category to contribute in the same way to the truth conditions of sentences in which they appear. This is a semantic constraint, and is not guaranteed by Tarski's syntactic criterion. So, the existence of a partitioning \(\tau\) that meets the criterion and also will serve these further purposes should really be read as a constraint on the sort of languages which the account will cover.

These variables are extra-linguistic relative to the object language, but this is not to say that they are meta-linguistic variables. When the non-logical terms of the language are replaced in a sentence by these special variables, the result is an open sentence in a hybrid object language.

The hybrid language \(L^*\) is an extension of the original \(L\) whose symbols are \(S^* = S \cup Z\) with semantic categories given by \(\tau^* = \tau \cup h\) (where \(h\) is the function on \(Z\) which takes \(z \in Z\) to \(\tau(F^{-1}(z))\)), and where the formation rules are extended from \(L\) in the obvious way.

It should be noted here that the sort of sequences needed and intended by Tarski will be a focus of debate in Section 3 of this paper.

Tarski discusses satisfaction at length in (Tarski, 1933 and 1944). The reader unfamiliar with Tarski's construction is referred to these works.

He offered such a criterion in a 1966 lecture (Tarski, 1986). Gila Sher (1991) gives an account of (logic and) logical terms which incorporates this criterion.

This is implicit throughout (Tarski, 1936), but see especially p. 416 where the notion of satisfaction is deployed.

Sher (1994) reads Tarski's two conditions as a necessity condition and a formality condition. She takes the first to be a modal condition, namely, necessary truth preservation. She adopts this stronger reading, in part at least, as a response to Etchemendian objections that mere truth preservation is not enough. Cf. Section 5.3, Section 4.2. Sher's formality condition is based on her own notion of formal properties, and will not be discussed here.

Though lacking an account of logical terms, Tarski points out that, when applied to a certain familiar class of languages with respect to which we think we can distinguish intuitively between logical and non-logical terms, the account gets the right answer (pp. 417–418).

I am indebted to Kirk Ludwig and Stewart Shapiro for helping me toward a better appreciation of the complex relationship between Tarski's definitions and the pre-theoretic notions.

'S\(_n\)0' is an abbreviation for a string of \(n\) many 'S' symbols followed by '0'. '\(\varphi[S_n0]\)' then signifies the result of substituting that string in for all free occurrences of '\(x\)' in formula \(\varphi\x\).

Tarski interprets Gödel incompleteness as showing that the general limitation revealed by \(\omega\)-incompleteness cannot be patched up with more deductive rules of inference. So, Tarski held that no proof-theoretic treatment of consequence could be adequate. Cf. (Tarski, 1936, p. 412).

Even if this is right, it is not clear whether Tarski thought that these should be treated as logical terms generally, or not. Since he allows (p. 420), somewhat reluctantly, that
the notion of a logical term may have to be treated as discourse-relative, it may be that only in the context of number theory would these arithmetical items be treated as logical terms.

17 Etchemendy says that there is a “fundamental conflict between the use of such restrictions [as it would be necessary to incorporate into the Tarskian account to prevent the divergence of the two accounts] and Tarski’s general account of the logical properties... The standard model theory for first-order languages seems to violate the very guidelines that underlie the [Tarskian] approach to semantics... [Though,] it is always possible that Tarski’s account of the logical properties can be suitably revised so that the standard cross-term restrictions used in first-order model theory turn out to be consistent with it” (p. 77). The implication, I take it, is that Tarski’s account unrevised is inconsistent with that which would bring it into compliance with the model-theoretic account. Thus, no mere refinement of Tarski’s account could do the job. That the divergence between the model-theoretic and Tarskian accounts provides cases in which the former is getting it right and the latter not will be evident from the nature of the divergence.

18 For simplicity, I speak here, as elsewhere, to the first-order case.

19 Let my own presentation of Tarski’s account not prejudice the issue. Recall that in my characterization of sequences, I have been more explicit than Tarski was in his 1936 paper.

20 I believe Corcoran may mistakenly perceive there to be a connection, because he is under a misapprehension about another historical matter which involves a misunderstanding of Tarski’s account. Cf. Note 32.

21 Hodges’ suggestion comports well with the story Carnap (1963) tells about the circumstances surrounding Tarski’s Paris lectures. In fact, Tarski seems to have considered the subject of quite specialized interest even among logicians. Where domain-relativization enters his highly technical (Tarski, 1933, p. 199 footnote 2), he suggests that this material be skipped by those “who are not interested in special studies in the domain of the methodology of the deductive sciences”.

22 It is interesting to note that Dag Prawitz (1985, p. 154) reports that in Bolzano’s account, which is closely related to Tarski’s, the relativization of quantifiers to domains was explicit (though I have not identified a passage in Bolzano which corroborates this).

23 My discussion of Tarski’s claim about material consequence follows Gila Sher’s (1991, p. 45) discussion, though her motivation for that discussion is quite different than mine here. It might be thought that she has offered a straightforward argument to the effect that Tarski made this mistake. However, it is important to see that Sher’s discussion presupposes that “Tarskian semantics” just is modern model-theoretic semantics — a presupposition that begs the historical question (though does not beg any questions with which Sher is there concerned).

24 There is, in fact, another way that Tarski’s claim can be understood to be true, namely, if the claim is restricted to propositional languages.

25 Otherwise it is hard to explain Tarski’s use of the obviously eliminable device of substituting extra-linguistic variables for the non-logical terms prior to evaluating formulas relative to sequences.

26 See (Tarski, 1933, Section 3, esp. pp. 199–200, 207). The authority for the following definition comes from (1933, p. 200). Seeing the opportunity to cast the technique in a more general light, I have made the clause for atomic formulas parasitic on the unrelativized notion of satisfaction, which Tarski earlier defines for the calculus of class-
es (p. 193), whereas Tarski's atomic clauses re-introduce the appropriate set-theoretic conditions. It is expository simpler to do as I have done, though not essential.

27 A $\chi$-variant of a sequence $s$ is a sequence that differs from $s$ in at most its $\chi$th position, i.e. on the value it takes on $f(\chi)$.

28 There are strong parallels here to Tarski’s method of approach to defining truth.

29 Let the terms ‘Substitution Condition’ and ‘Substitutional Necessity’ ambiguously refer to one of these pairs. In subsequent discussion, context will resolve which is meant.

30 Or so it is fabled in philosophic folklore, thanks to Quine.

31 Cf. (Etchemendy, 1990, pp. 162–163, footnotes 2, 5). In (Etchemendy, 1983, pp. 326, 328), the same misattribution is made, but not flagged.

32 The connection between his own definition and Bolzano's is acknowledged in a footnote to that definition added by Tarski to the 1983 translation of his 1936 essay (p. 417). I do not pretend to any thoroughgoing study of Bolzano’s work, and there may yet be room for re-interpretation of Bolzano’s notion of idea. I will rest content to note that what I have said here also accords with what is said in (Kneale, 1962) both about Bolzano’s account, and its relation to Tarski’s. It likewise comports with Rolf George’s view of Bolzano (see the editor’s introduction to (Bolzano, 1837, p. xxxiv), and (George, 1972, p. 116)). Though I think George’s (1972, p. 113) presentation of what is, in effect, [EB$_1$] as “Tarski, but in a somewhat less technical vein” is most unfortunate. Corcoran (1973, p. 71), however, seems clearly to have mistaken [EB$_1$] for Bolzano’s account, and then mistakenly identified Tarski’s account with this (p. 70, line 3, notwithstanding).

33 This characterization of the idea is adequate for our purposes, even though it would certainly need refinement if we were going to work with it at all closely. For example, if we took a classical first-order language with no empty names and extended it by adding some empty names, this might well make a difference in the logical status of sentences like ‘$\forall xFx \rightarrow Fa$’ as these occur in the two languages, even though this particular constant, ‘$a$’, refers.

34 I shall write ‘some\thing’ to indicate this logical parsing. Logically, the move is tantamount to treating quantifiers as always restricted by a common noun, so that ‘some\thing’ and ‘some person’ may be formalized uniformly as ‘[some $x$: thing $x$]’ and ‘[some $x$: person $x$]’, respectively.

35 For example, ‘Every\thing is busy’ will not be in the substitution class of ‘Some\thing is busy’.

36 I am responding in terms of the example with which I epitomized Etchemendy’s incompatibility argument, but my point generalizes (just as Etchemendy’s incompatibility argument does).

37 While Etchemendy talks about “truth values” here, it is clear from the context that he must mean truth-in-a-sequence not truth simpliciter. In fact, we might want to phrase the principle being implicitly invoked in terms of satisfaction instead of truth of any kind. We would need this more general notion anyway if we were to follow out Etchemendy’s suggestion.

38 These terms are only in effect given certain semantic values by sequences, because, literally speaking, only variables (linguistic and extra-linguistic) are assigned values by sequences.

39 The semantic clause for conjunction induces a function which could naturally be thought of as the extension with respect to which ‘and’ is evaluated. In a purely sentential language, the function induced is simply a function from pairs of truth values to truth values. For quantified languages the situation gets a bit more complex, but roughly, the
extension could be taken to be the function which takes a pair of sets of sequences to the set-theoretic intersection of that pair.

Incidentally, Etchemendy seems to construe Tarski’s references to the “ordinary notion of logical consequence” so as to come to something like “the ordinary person’s notion of what follows from what”. But I think it is clear that Tarski had in mind the concept of logical consequence employed by logicians and philosophers. Persons not trained to it do not have a distinguished notion of logical (i.e. formal) consequence in Tarski’s sense.

Cf. (Etchemendy, 1988, p. 60, footnote 8).

Strictly speaking, our sentence is supposed to be of the form \( \neg \exists x_1 \exists x_2 \cdots \exists x_n x_1 \neq x_2 \land x_2 \neq x_3 \land \cdots \land x_{n-1} \neq x_n \land x_n \neq x_1 \).

Again, we need not commit ourselves to the existence of infinitely many things, if our talk of at least countable infinities admits of a finitistic treatment. In particular, we need to underwrite an infinitary model theory. Thus, Chihara’s (1990) modal-nominalist treatment of mathematics could be used to this end, and indeed, Chihara (forthcoming) suggests it.

Cf. (Etchemendy, 1990, p. 87).

And Etchemendy does not give us a clear characterization. For some acerbic remarks on this matter, see (Hart, 1991, p. 492).

My own suspicion, for what it is worth, is that Tarski’s account already has the requisite modal strength (to support [9s], but not [cf]) in virtue of the fact it meets the adequacy condition, [Adequacy]. That would want showing, of course. This position is similar to that held by Sher, however, she argues her case on the basis of a reading of Tarski’s adequacy conditions that I think incorrect.

Here again, [Co-extensiveness] is assumed by Etchemendy as a condition of adequacy for the account.

Etchemendy follows Tarski in mistakenly supposing that fixing all the terms of the language will reduce F-consequence to material consequence. We discussed this error, which was first pointed out by Sher, earlier. It is a natural mistake for Etchemendy to make, because he believes (falsely, I have argued) that Tarski was not using, and could not have consistently used domain-relativization. If that were true, the above argument would have turned out to be valid.

Etchemendy initially intimates (p. 86), but falls just short of explicitly claiming, that Tarski offered the following result so as to establish [8]. However, Etchemendy’s ensuing critical line relies on the idea that Tarski offered the result in the service of proving [8R], and hence [8].

It is prima facie unlikely that Tarski would have thought it necessary to prove something like [9s], since we know that at least by 1940, and perhaps as early as 1930, he argued that logical truth differed only in degree from factual truth. I take this to indicate that Tarski does not think of logical truth (or, presumably, the other logical properties) as a modal notion. Cf. (Carnap, 1963, pp. 30, 35–36). This point is made by W.D. Hart (1991) in his review of Etchemendy’s book.

The scare quotes here are to indicate that, rather than appealing to the notion of Tarskian consequence defined back in Section 1.1, we mean here a notion which Etchemendy believes to be Tarski’s notion, and which belief, I will argue in Section 5.2, seems to be the result of a quantifier scope error. Etchemendy’s characterization would have it that \( \varphi \) is a “Tarskian consequence” of \( \Gamma \) iff for some \( F \), \( \varphi \) is an F-consequence of \( \Gamma \).
John Peoples correctly observes that, strictly speaking, we only need the (apparently) weaker claim: if for some \( F \), \( \varphi \) is an \( F \)-consequence of \( \Gamma \) and it is necessary that [if every sentence in \( \Gamma \) is true, then \( \varphi \) is true], then \( \varphi \) is a logical consequence of \( \Gamma \). However, as Etchemendy points out in his argument, the first conjunct of this extended antecedent is trivial to satisfy. In fact, if Etchemendy’s argument for [8L] were sound, then the first conjunct of the antecedent here would be entailed by the second conjunct. So, this claim would not be weaker after all, by Etchemendy’s lights.

Etchemendy comes close to saying as much (p. 82).

A referee for this Journal points out that Etchemendy (p. 82) expresses some doubt about [10], but notice that this might at best suggest that Etchemendy was unaware that his argument committed him to [10].

I assume here that definitions of logical truth and logical consistency in terms of logical consequence would be unproblematic. This justifies the present implicit use of a simplification of [Co-extensiveness].

There is also a case where a temporal locution is used. Tarski says, “Consider any class \( K \) of sentences and a sentence \( X \) which follows from the sentences of this class. From an intuitive standpoint it can never happen that both the class \( K \) consists only of true sentences and the sentence \( X \) is false” (Tarski, 1936, p. 414). I don’t suppose anyone would take seriously the suggestion that Tarski would prove this in temporal logic. No, his usage has a familiar colloquial structure.

It is worth noting that all of the theorems just mentioned may be proved uniformly in \( L \), which means, roughly, that the result is proved in the same way for all languages in \( K \). I mention this just to point out that a revised “bogus consequence” relation that was only shown to satisfy (1)–(5) in a way that depended essentially on the particulars of some chosen target language would, it seems to me, be criticizable also on these grounds.

In personal correspondence, Gary Curtis suggests that what Etchemendy is worried about is “epistemic circularity”. This is suggested by Etchemendy’s claim that “in general, it will be impossible to know whether an argument [meets the conditions for Tarskian consequence] without antecedently knowing the specific truth values of its constituent sentences”. Curtis suggests that Etchemendy’s reasoning goes something like this:

1. Suppose every sentence in \( K \) is true. (p)
2. In order to know that \( S \) is a Tarskian consequence of \( K \), one must know the specific truth value of \( S \) (and perhaps also the truth values of the sentences in \( K \)). (p)
3. If \( S \) is a Tarskian consequence of \( K \), then \( S \) is true. (1, Corollary C)
4. In order to know that \( S \) is a Tarskian consequence of \( K \), one must know that \( S \) is true. (2, 3)
5. Thus, knowing that \( S \) is a Tarskian consequence of \( K \) can provide no epistemic warrant for [believing that] \( S \) [is true]. (4)
6. However, knowing that \( S \) is a logical consequence of \( K \) can provide epistemic warrant for [believing that] \( S \) [is true]. (p)
7. Therefore, Tarskian consequence is not logical consequence. (5, 6)

However, if this is Etchemendy’s argument, it is surely not a good one. I can see no reason to accept premise (2), and I can think of good reasons not to. For example, I could come to know that \( S \) is a Tarskian consequence of \( K \) on the basis of deductive proof, together with a soundness argument for my proof system. This would not require that I know the truth values of any of the sentences of \( K \), or \( S \).
McGee’s argument concludes with a sentence of the right form, but, by his own admission, the only way to construe the argument so that it is sound gives that conclusion the wrong modal strength. So, I take it that McGee offers his construction as a sort of “partial result”.

It is well to remind ourselves that in 1936 even Tarski was not absolutely committed to there being a univocal fixing of logical terms. Though I suppose his 1966 characterization of logical notions would ultimately suggest that such relativization would not be appropriate (Tarski, 1986).

For simplicity, [EQ] is stronger than strictly necessary. For example, it is only conditioned on the identity of the truth conditions of the target sentences, even though I have, in setting up the argument, specified enough to determine the meanings of the sentences. My argument would work as well for any such weakening of the condition.

Thus, my criticism of Etchemendy’s claim will not rest on attributing to him the detailed argument which I hazarded above on his behalf. Rather, my criticism will stand provided that Etchemendy’s argument commits him to [X] and [EQ]. (Even this is somewhat too general. Cf. Note 61.)

In a session of the APA where I presented my reductio of the “bullet” argument, Etchemendy allowed that, in his view, “all bachelors are unmarried” is a logical truth. It also appears that Etchemendy (1990, p. 151) takes some truths of arithmetic which are not model-theoretically valid to be logical truths. Cf. (Chihara, forthcoming, Section 3). Chihara gives a further example: ‘(x)(y)(Lxy → Rxy)’ is given as an example of a logical truth in Barwise and Etchemendy’s textbook (1990, p. 102).

We also have the indirect evidence of Section 5.1 where we uncovered a conflation of logical consequence with necessary implication. Also, in (Etchemendy, 1983, p. 321), Etchemendy tells us that “there remain as many sentences and arguments that do not submit to formal treatment as sentences and arguments that do. Thus, our logical intuitions tell us that ‘Tom is a brother’ entails ‘Tom is a sibling’, that the latter is a consequence of the former. But our grammatical intuitions preclude the formal treatment of this argument, since many arguments with the same intuitive structure are obviously invalid” (first emphasis mine).

There is some finicky business here regarding how to treat the variable symbol that appears right after the quantifier. (The other occurrence of the variable must be treated as a non-logical term.) We can finesse the problem by clever choice of notation.

Tarski’s account of the logical properties is an extension of his satisfaction-based account of truth, and this latter is defined only over meaningful and semantically precise languages. So, the non-logical terms all have definite extensions. Recall also, that sequences were defined relative to a fixed choice of ordering function, f, which assigns a unique natural number to the terms of Z ∪ varL.

L* gives what Etchemendy calls (p. 56) a “semantically well-behaved reinterpretation” of L.

We do not have the same problem with the statement of Theorem F, since the quantifier over languages there is a suitably restricted quantifier. Roughly, for any specification of the parameter, L, Theorem T quantifies only over languages of expressive power equalling that of L (and hence, less than that of the metalanguage).
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